Proceeding of International Conference on Electrical Engineering, Computer Science and Informatics (EECSI 2014), Yogyakarta, Indonesia, 20-21 August 2014

3D-FDTD Method for Analysis of Rectangular Waveguide Loaded with Anisotropic Dielectric Material

Maulana Randa[1,2*] and Achmad Munir[2†]
[1]Department of Research and Development, The Ministry of Defense, The Republic of Indonesia
[2]Radio Telecommunication and Microwave Laboratory
School of Electrical Engineering and Infomatics, Institut Teknologi Bandung, Indonesia
*randa@students.itb.ac.id, †munir@ieee.org

***Abstract*—One of the most popular techniques to solve electromagnetic problems numerically is using finite-difference time-domain (FDTD) method. The method has been successfully applied to an extremely wide variety of electromagnetic problems. The essential reason resides in the fact that the FDTD method itself is extremely simple even for analyzing in a three-dimensional (3D) system. In this paper, the analysis of resonant frequency for a rectangular waveguide which is loaded with anisotropic dielectric material is numerically investigated based on 3D-FDTD method. The wave equations and modes that appear in the waveguide are analyzed theoretically in which the results are applied to validate the numerical result obtained from 3D-FDTD method. For comparison, an empty rectangular waveguide and a rectangular waveguide fully loaded with isotropic dielectric material are also analyzed both theoretically and numerically. From the result, it shows that a good agreement has been achieved between theoretical calculation and 3D-FDTD numerical results with their discrepancies of 0.26–2.32%.**

***Keywords*—3D-FDTD method; anisotropic dielectric material; rectangular waveguide; resonant frequency**

I. Introduction

Over the last decade, the analysis of electromagnetic wave propagation inside of waveguide which is fully or partially loaded with some dielectric materials has been the research subject for many researchers involved in electromagnetic research community [1]– [4]. Several numerical approximation methods have been utilized to obtain the answer of electromagnetic problems in the structure as well as for others. Some of the numerical methods solve the problem in time domain [4]– [5], whilst the others analyze in frequency domain [6]– [8]. In order to achieve more accurate results, although it usually requires more time and computer resources for solutions, all field effects should be included in full-wave numerical methods. Therefore, an alternate approach by taking a direct solution to Maxwell's time-dependent curl equations has been proposed to solve the electromagnetic problem of the structure in the time-domain, namely finite-difference time-domain (FDTD) method [9]– [10].

The idea taken in the FDTD method is to simplify in discretizing Maxwells equations, both in space and time, with central difference approximations. The main point to utilize this method is by applying Yee's algorithm [11]. The originality of Yee's algorithm resides on the allocation of electric and magnetic field components in space and the series in time for the evolution of procedure. In the implementation, the space that covers all the source or the scatterer should be firmly determined to be divides into a number of small cell, called as Yee's cell. Then, the solution of electric and magnetic field components for each cell is obtained by solving the Maxwell's equations with appropriate boundary conditions.

In this paper, a 3D-FDTD method is employed for analyzing the resonant frequency of rectangular waveguide which is loaded with anisotropic dielectric material. The FDTD numerical analysis of resonant frequencies is carried out by deriving Maxwell's equations for anisotropic dielectric material inside of waveguide and applying proper boundary conditions for the walls of rectangular waveguide. Here, the dielectric material which has anisotropic permittivities is defined by making a different relative permittivity for each direction in 3D Cartesian coordinate system [12]. To validate the numerical result obtained from 3D-FDTD method, the calculation of resonant frequency is also performed theoretically and its result is presented consecutively.

II. Brief Theory of Anisotropic Dielectric Material and 3D-FDTD Method

For analyzing the resonant frequency of rectangular waveguide loaded with anisotropic dielectric material illustrated in Fig. 1, the permittivity and permeability of anisotropic dielectric material are expressed in (1) [12].

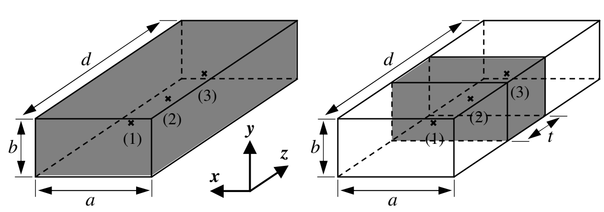

Fig. 1. Rectangular waveguide loaded with anisotropic dielectric material (× is an observation point); fully loaded (left), partially loaded (right)

This work is partially supported by the Directorate General of Higher Education (DGHE), the Ministry of Education and Culture, the Republic of Indonesia, under the program scheme of decentralization research 2014.

$$[\epsilon] = \epsilon_0 \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}, \qquad [\mu] = \mu_0 \tag{1}$$

where ϵ_x, ϵ_y and ϵ_z are the relative permittivity in the x, y and z directions, respectively. Whilst, ϵ_0 and μ_0 are permittivity and permeability in free space, respectively. The field in the dielectric material inside of rectangular waveguide is assumed to propagate in the $+z$ direction as $e^{(j\omega t - j\beta z)}$. By substituting (1) into Maxwell's equations given in (2)-(3) and solving them with respect to H_z for the TE wave mode, all other fields inside of rectangular waveguide, i.e. E_x, E_y, H_x and H_y, can be obtained.

$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H} \tag{2}$$

$$\nabla \times \mathbf{H} = j\omega[\epsilon]\mathbf{E} \tag{3}$$

By applying proper boundary conditions for the walls of rectangular waveguide with the width of a, height of b and length of d, the formulation of resonant frequency (f_0) for rectangular waveguide fully loaded with anisotropic dielectric material is expressed in (4).

$$f_{0,nml} = \frac{c}{2\pi\sqrt{\epsilon_y}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \frac{\epsilon_y}{\epsilon_x}\left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \tag{4}$$

where c is speed of light in free space and n, m, and l are the number of field variations in the x, y and z directions, respectively. For rectangular waveguide loaded with isotropic dielectric material and empty or hollow rectangular waveguide, the formulations of resonant frequency can be obtained by replacing ϵ_x and ϵ_y in (4) with ϵ_r and 1 as expressed in (5) and (6), respectively.

$$f_{0,nml} = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \tag{5}$$

$$f_{0,nml} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \tag{6}$$

Since the above formulations , i.e. (4) and (5) are addressed for rectangular waveguide fully loaded with dielectric material, to calculate the resonant frequencies of rectangular waveguide partially loaded with dielectric material, therefore, it can be obtained by using (7).

$$f'_{0,nml} = \frac{f_{0,nml}}{\sqrt{\epsilon_{eq}}} \tag{7}$$

where ϵ_{eq} is defined as the equivalent relative permittivity of dielectric material inside of rectangular waveguide expressed in (8), ϵ_1 is the relative permittivity of dielectric material, V_{dm} and V_{rg} are the volume of dielectric material and rectangular waveguide, respectively.

$$\epsilon_{eq} = 1 + \frac{V_{dm}}{V_{rg}}(\epsilon_1 - 1) \tag{8}$$

The implementation of 3D-FDTD method in the analysis of resonant frequency for rectangular waveguide fully or partially loaded with anisotropic or anisotropic dielectric material is carried out by discretizing, both in space and time domain, each component of electric and magnetic fields in (2) and (3). For the TE wave mode, the FDTD notation is expressed in (9)–(10) and (11)–(13) for electric and magnetic fields component, respectively [13]. By using the electric and magnetic fields component placement as illustrated in Fig. 2, the explicit finite-difference approximation for (9)–(13) in 3D Cartesian coordinate system can be obtained.

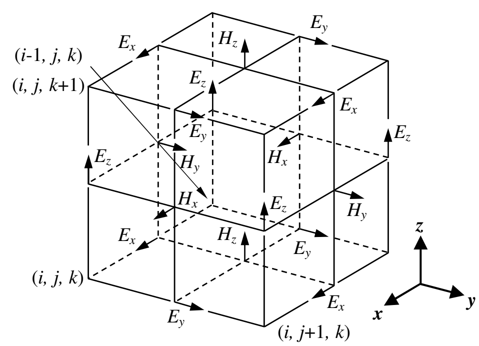

Fig. 2. Fields component placement in 3D Cartesian coordinate system

$$\frac{E_x\big|^{n+1}_{i+\frac{1}{2},j,k} - E_x\big|^{n}_{i+\frac{1}{2},j,k}}{\Delta t} = \frac{H_z\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\epsilon_0\epsilon_{i+\frac{1}{2},j,k}\Delta y} - \frac{H_y\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\epsilon_0\epsilon_{i+\frac{1}{2},j,k}\Delta z} \tag{9}$$

$$\frac{E_y\big|^{n+1}_{i,j+\frac{1}{2},k} - E_y\big|^{n}_{i,j+\frac{1}{2},k}}{\Delta t} = \frac{H_x\big|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x\big|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\epsilon_0\epsilon_{i,j+\frac{1}{2},k}\Delta z} - \frac{H_y\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_y\big|^{n+\frac{1}{2}}_{i-\frac{1}{2},j+\frac{1}{2}.k}}{\epsilon_0\epsilon_{i,j+\frac{1}{2},k}\Delta x} \tag{10}$$

$$\frac{H_x\big|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x\big|^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}}{\Delta t} = \frac{E_y\big|^{n}_{i,j+\frac{1}{2},k+1} - E_y\big|^{n}_{i,j+\frac{1}{2},k}}{\mu_0\Delta z} \tag{11}$$

$$\frac{H_y\big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y\big|^{n-\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}}{\Delta t} = -\frac{E_x\big|^{n}_{i+\frac{1}{2},j,k+1} - E_z\big|^{n}_{i+\frac{1}{2},j,k}}{\mu_0\Delta z} \tag{12}$$

$$\frac{H_z\big|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_z\big|^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}}{\Delta t} = \frac{E_y\big|^{n}_{i+\frac{1}{2},j+1,k} - E_y\big|^{n}_{i+\frac{1}{2},j,k}}{\mu_0 \Delta y} - \frac{E_z\big|^{n}_{i+1,j+\frac{1}{2},k} - E_z\big|^{n}_{i,j+\frac{1}{2},k}}{\mu_0 \Delta x} \quad (13)$$

III. Result and Discussion

There are 4 scenarios of rectangular waveguide that will be analyzed using 3D-FDTD method; (i) empty or hollow rectangular waveguide; (ii) rectangular waveguide fully loaded with isotropic dielectric material (ϵ_r = 4.5); (iii) rectangular waveguide fully loaded with anisotropic dielectric material (ϵ_x = 4.5, ϵ_y = 9, and ϵ_z = 4.5); (iv) rectangular waveguide partially loaded with anisotropic dielectric material (ϵ_x = 4.5, ϵ_y = 9, and ϵ_z = 4.5). The dimension of rectangular waveguide for all scenarios is 40mm width (a), 20mm height (b), and 100mm length (d). Whilst the thickness of anisotropic dielectric material (t) in the 4th scenario is 10mm. The Gaussian pulse modulated by continuous wave is applied as an incident wave to excite the TE_{10} wave mode. In each scenario, the time-domain data is picked up through 3 observation points which are located inside of waveguide, i.e. (20mm, 10mm, 25mm), (20mm, 10mm, 50mm) and (20mm, 10mm, 75mm).

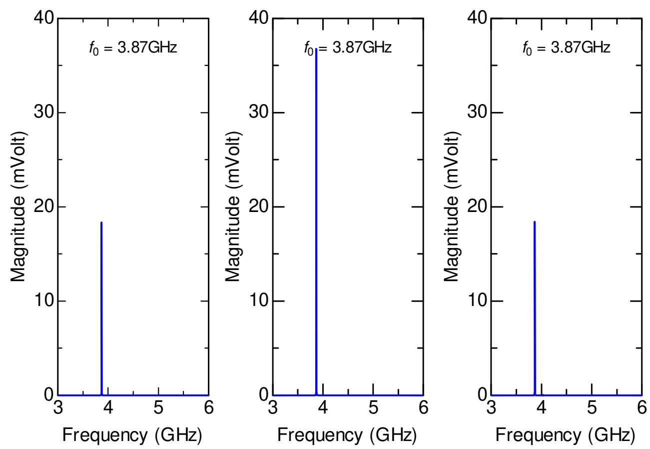

Fig. 3. Resonant frequency of empty rectangular waveguide; 1st observation point (left); 2nd observation point (center); 3rd observation point (right)

By applying fast Fourier transform (FFT), the time-domain data in each observation point is then transformed into frequency-domain data to analyze the resonant frequency response. The result of FFT is plotted in Figs. 3, 4, 5 and 6 for the 1st, 2nd, 3rd and 4th scenario, respectively. To validate the numerical result obtained from 3D-FDTD method, the calculation is performed theoretically in which the calculation results are summarized in Table I with the 3D-FDTD numerical result tabulated together for comparison.

From the result shown in Figs. 3–6, it is noticeable that the 3D-FDTD method can accurately analyze the resonant frequency for rectangular waveguide in all scenarios. However, the resonant frequency of higher order mode for rectangular waveguide fully loaded with isotropic dielectric material indicated in Fig. 4 also appears beside the lowest TE_{10} wave mode. This is probably evoked by the value of ϵ_r which is low, i.e.

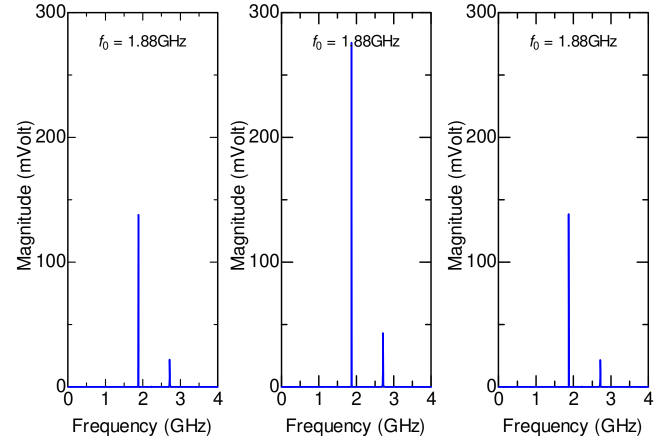

Fig. 4. Resonant frequency of rectangular waveguide fully loaded with isotropic dielectric material (ϵ_r = 4.5); 1st observation point (left); 2nd observation point (center); 3rd observation point (right)

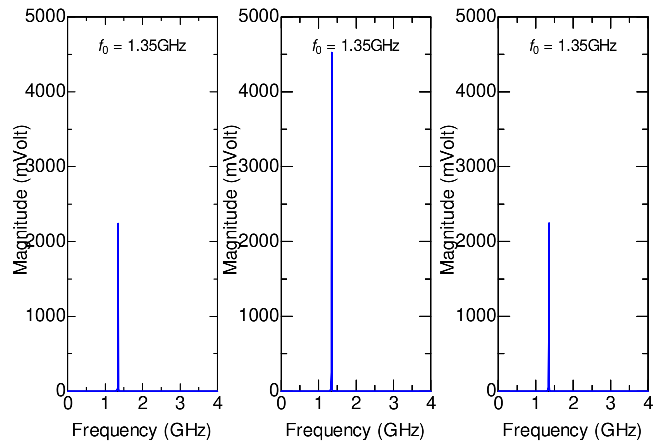

Fig. 5. Resonant frequency of rectangular waveguide fully loaded with isotropic dielectric material (ϵ_x = 4.5, ϵ_y = 9, and ϵ_z = 4.5); 1st observation point (left); 2nd observation point (center); 3rd observation point (right)

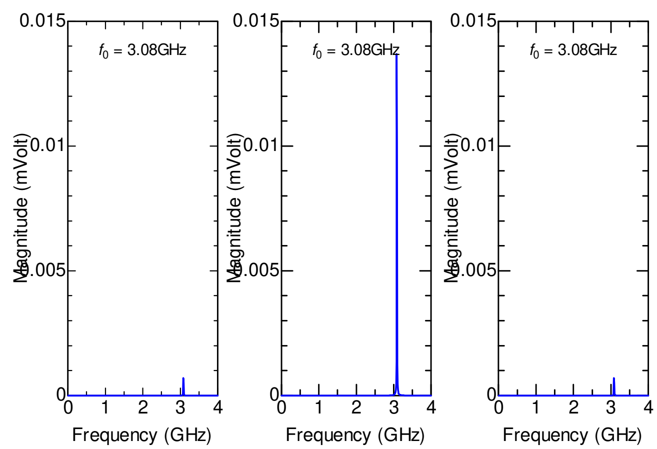

Fig. 6. Resonant frequency of rectangular waveguide partially loaded with isotropic dielectric material (ϵ_x = 4.5, ϵ_y = 9, and ϵ_z = 4.5); 1st observation point (left); 2nd observation point (center); 3rd observation point (right)

4.5, while the bandwidth of Gaussian pulse excitation wave is too wide affecting the excitation of higher order mode. Furthermore, from Table I, it shows that the 3D-FDTD numerical results have good agreements with the theoretical calculation with the discrepancies between them of 0.26–2.32%. The widest discrepancy, i.e. 2.32%, is produced by the rectangular waveguide partially loaded with anisotropic dielectric material which is possibly caused by the discontinuity of material that occurs in the waveguide. Although it still needs to be proven by simulation, the discrepancy can be minimized by reducing the cell size especially in the discontinuity area.

TABLE I. COMPARISON OF THEORETICAL AND 3D-FDTD NUMERICAL RESULTS FOR ALL SCENARIOS

Scenario	**Theoretical result (GHz)**	**3D-FDTD method (GHz)**	**Discrepancy**	
			(MHz)	*(%)*
1st scenario	3.86	3.87	10	0.26
2nd scenario	1.90	1.88	20	1.05
3rd scenario	1.34	1.35	10	0.75
4th scenario	3.01	3.08	70	2.32

IV. CONCLUSION

The analysis of resonant frequency for a rectangular waveguide loaded with anisotropic dielectric material has been investigated using 3D-FDTD method. The theoretical approach has also been presented to validate the numerical result obtained from 3D-FDTD method. To verify the accurateness of 3D-FDTD method, the resonant frequency analysis of an empty rectangular waveguide and of a rectangular waveguide fully loaded with isotropic dielectric material has been numerically performed and also confirmed with the theoretical approach. From the result, it has demonstrated that the 3D-FDTD method has accurately analyzed the resonant frequency of waveguide structure in which the 3D-FDTD numerical results have coincided with the theoretical calculation results. The discrepancies between theoretical calculation and 3D-FDTD numerical results were 0.26–2.32% with the widest discrepancy produced by a rectangular waveguide partially loaded with anisotropic dielectric resonator. Therefore, it can be concluded that the 3D-FDTD method is very effective to solve electromagnetic problems for some structure contains anisotropic dielectric material.

ACKNOWLEDGMENT

The authors would like to thank Dr.-Ing. Chairunnisa and Dr. Mohammad Ridwan Effendi both from the School of Electrical Engineering and Informatics, Institut Teknologi Bandung (ITB) for strong support and fruitful discussion.

REFERENCES

[1] M. Samardzija, J. Hirokawa and M. Ando, "Scattering analysis of H-plane T-junction between fully dielectric-filled rectangular waveguide and partially dielectric-filled parallel-plate," in *Proceeding of IEEE Antennas and Propagation Society International Symposium (AP-S) 2008*, San Diego, CA, Jul. 2008, pp.1–4.

[2] R. Zhong, Q. Zheng, J. Peng, B. Yao, W. Xu, T. Xiang and L. Li, "Analysis of inhomogeneously filled cavities by vector finite element method," in *Proceeding of 2nd International Conference on Mechanic Automation and Control Engineering (MACE) 2011*, Hohhot, China, Jul. 2011, pp. 1036–1039.

[3] R. Abdullin, S. Knyazev, L. Lesnaya, and S. Shabunin, "Analysis of partially dielectric-filled rectangular waveguide with transverse slots using Green's function method," in *Proceeding of 7th European Conference on Antennas and Propagation (EuCAP) 2013*, Gothenburg, Sweden, Apr. 2013, pp. 3570–3574.

[4] J. Liu, D. R. Jackson and Y. Long, "Modal analysis of dielectric-filled rectangular waveguide with transverse slots," *IEEE Trans. Antennas Propag.*, Vol. 59, Issue 9, pp. 3194–3203, Sep. 2011.

[5] I. A. Eshrah, A. A. Kishk, A. B. Yakovlev and A. W. Glisson, "Modal analysis of corrugated rectangular waveguides supporting left-hand propagation," in *Proceeding of IEEE Antennas and Propagation Society International Symposium (AP-S) 2005*, Washington, USA, Jul. 2005, pp. 664–667.

[6] A. Munir, H. Kubo, A. Sanada and I. Awai, "2-D finite-difference frequency-domain method and its application for dispersion characteristic analysis of ferrite devices," *Microwave and Optical Tech. Lett.*, Vol. 41, No. 6, pp. 437–439, Jun. 2004.

[7] Y. Liu, S. Safavi-Naeini and S. K. Chaudhuri, "Determination of resonant modes of dielectric resonators using MoM-SIE with combined entire-domain and subdomain basis functions," *IEEE Trans. Antennas Propag.*, Vol. 53, Issue 2, pp. 883–886, Feb. 2005.

[8] P-J. Chiang, C-L. Wu, C-H. Teng, C-S. Yang and H-C. Chang, "Full-vectorial optical waveguide mode solvers using multidomain pseudospectral frequency-domain (PSFD) formulations," *IEEE J. Quantum Electron*, Vol. 44, Issue 1, pp. 56–66, Jan. 2008.

[9] K. L. Tsakmakidis, C. Hermann, A. Klaedtke, C. Jamois and O. Hess, "Systematic modal analysis of 3-D dielectric waveguides using conventional and high accuracy nonstandard FDTD algorithms," *IEEE Photon. Technol. Lett.*, Vol. 17, Issue 12, pp. 2598–2600, Dec. 2005.

[10] A. Sanada, K. Okubo and I. Awai, "Full-wave finite-difference time-domain formulation for gyromagnetic ferrite media magnetized in arbitrary direction," *IEICE Trans. Electron.*, Vol. E84-C, No. 7, pp. 931–936, Jul. 2001.

[11] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equation in isotropic media," *IEEE Trans. Antennas Propag.*, Vol. AP-14, Issue 3, pp. 302–307, May 1966.

[12] A. Munir, N. Hamanaga, H. Kubo and I. Awai, "Artificial dielectric rectangular resonator with novel anisotropic permittivity and its $TE_{10\delta}$ mode waveguide filter application," *IEICE Trans. Electron.*, Vol. E88C, No. 1, pp. 40-46, Jan. 2005.

[13] A. Taflove and S. C. Hagness, *Computational electrodynamics: The finite-difference time-domain method*, Artech House, Inc., 2005.