# Contouring Performance Study of a Fuzzy-based Practical Controller on an X-Y Table

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*Abstract*— This article presents a contouring performance of an improved practical fuzzy-based nominal characteristic trajectory following (NCTF) controller. The proposed fuzzy-based NCTF controller structure for continuous motion control is slightly different from the normal NCTF controller. It is composed of a nominal characteristic trajectory (NCT) and a fuzzy compensator designed using the practical design approach. Therefore, simple and easy of controller design are maintained. The NCT constructed through establishing procedure and the Mamdani type fuzzy compensator is designed according to the available information provided by NCT and the hardware specification. The contouring performance was evaluated by measuring the response of the system providing circle shape through simulation. The simulation result shows that the controller is promising as a practical controller for continuous motion.

*Keywords—NCTF; contouring; x-y table; fuzzy; practical controller*

## I. INTRODUCTION

Two-axes positioning systems are widely used in industrial applications such as for a feed drive system of the machine tools to carry workpieces or tools to the desired location. Their positioning accuracy determines the quality of the machine tools. Stud welding machines, milling machines and robotic systems are few to mention of such applications. Currently, the most frequently used in the feed drive system is the ball-screw drives [1]. Stacking two ball-screws to get perpendicular axes is the most common design. However, due to limitations of the conventional ball-screw feed drives, a good controller is required. The classical PID controller provides the simplest and yet the most efficient solution to many control problems [2]. However, the complexity of the macro to the micro-region model is increasing due to their different behavior. Hybrid, two-step approaches and feedback control with the disturbance observer [3, 4, 5] are proposed and fuzzy control provides an advantage in practical approach [6] and unknown model of the controlled object [7]. Moreover, the practical controller design method is also required for practical applications.

For practical applications, a nominal characteristic trajectory following controller had been proposed as a practical controller [8, 9]. The advantages of this controller are their simple structure and easy design procedure. Its structure consists of a nominal characteristic trajectory (NCT) and a PI compensator. In order to construct the NCT, a simple open loop experiment is conducted. From the NCT, PI compensator parameters are taken. However, an unlimited PI parameters combination is available from the stable region, thus designer's judgment or try and error approach is needed. With the intention to keep the NCTF controller simple, easier and more practical, NCTF controller with fuzzy compensator had been proposed [10]. The fuzzy compensator replaced PI compensator. NCTF controller with fuzzy compensator is effective for point-to-point positioning task, since it gives a similar response compared with the original NCTF controller. However, performances of the NCTF controller with a fuzzy compensator in contouring tasks require deep investigations. This study presents the performance investigation of the NCTF controller with a fuzzy compensator dealing with contouring tasks.

## II. NCTF CONTROLLER

The concept and design procedure of the NCTF control system have been explained in [7, 8, 9] and can be comprised into three steps: (i) Displacement and velocity responses of a stepwise input are measured, (ii) The NCT is constructed on the phase-plane using the displacement and velocity information during the deceleration range, and (iii) The compensator is then designed using the open-loop data and the NCT information.

The structure of the proposed fuzzy-based NCTF control system for continuous motion consists of an NCT and a fuzzy compensator is shown in Fig. 1(a) and the typical NCT of a controlled object is shown in Fig. 1(b). Important NCT parameters which are required for compensator design are the maximum error rate, $h_{max}$, the maximum error at NCT start, A, and the gradient at origin, m.

### A. Fuzzy Compensator

The structure of the fuzzy compensator is shown in Fig. 2. The fuzzy compensator is a Mamdani type fuzzy compensator with two inputs, $u_p$ and $u_i$ which is the integral of $u_p$. The output is the control signal, u.

Fig. 1. (a) The structure of NCTF control system with fuzzy compensator for continuous motion, (b) Typical NCT of a controlled object.



Fig. 2. The structure of the Mamdani type fuzzy compensator.
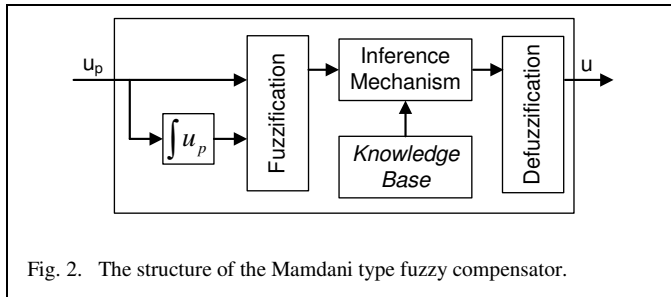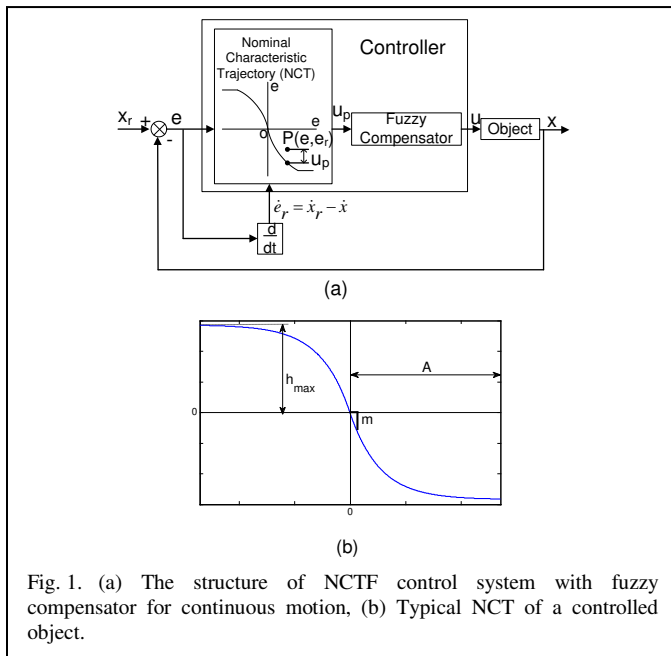
Construction of the rule base is designed according to the object motion in reaching and following NCT as shown in Fig. 3. The reaching phase region is when the object motion approaches NCT and the following phase region is when the object follows NCT within bounded specified accuracy. The fuzzy rules are summarized in Table 1.
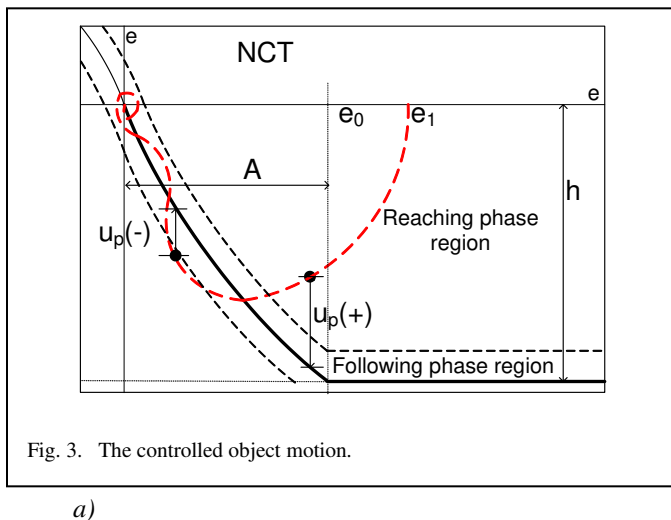


Fig. 3. The controlled object motion.

*a)*

TABLE I.    FUZZY RULE BASE

| | | $u_i$ | | |
|---|---|---|---|---|
| | | *N* | *Z* | *P* |
| | *N* | *N* | *N* | *Z* |
| $u_p$ | *Z* | *N* | *Z* | *P* |
| | *P* | *Z* | *P* | *P* |

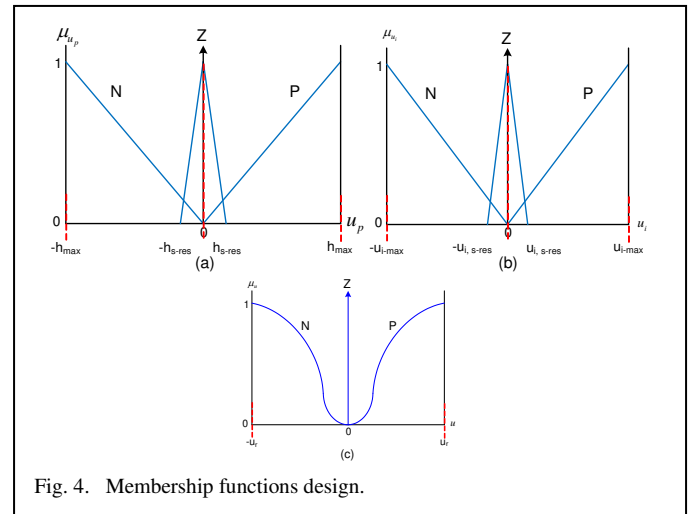## B. Membership Function Design

The fuzzification process converts crisp signal up and ui into fuzzy variables. The membership function of input up varies in the range of ±hmax as shown in Fig. 4(a). There are three triangular-shaped membership functions which are Negative (N), Zero (Z) and Positive (P). The membership function of up Zero (Z) has the value of error rate at resolution sensor, $h_{s-res}$. The membership function of input $u_i$ is shown in Fig. 4(b) consists of three triangular-shaped membership functions which are N, Z and P. The range of this input is ±$u_{i-max}$ calculated based on the following equation:

$$u_{i-max} = \int_0^A u_p de \cong 0.5 A h_{max} \qquad (1)$$

In the following phase, object motion oscillates within ± resolution sensor. Thus the membership function of $u_i$ (Z) can be simplified based on the following equation:

$$u_{i_{s-res}} = \left| \pm a_{s-res} \right| \left| \pm h_{s-res} \right| = 4 a_{s-res} h_{s-res} \qquad (2)$$



Fig. 4. Membership functions design.

The defuzzification process converts fuzzy variables into crisp control signal u. The membership function of output u is shown in Fig. 4(c). There are three membership functions which are z-shaped N, singleton Z and s-shaped P. The range of fuzzy variable output is ±ur, which is the rated motor input.

## III. THE CONTROLLED OBJECT

A series of simulation is carried out based on the dynamic model of the experimental two-axes linear positioning system as a controlled object as shown in Fig. 5(a). The X-axis mechanism is which the Y-axis mechanism attached to it and 1 µm sensor resolution is used. Dynamic model of the system is derived according to the information from NCT. Simplified model of the system is the following [8],

$$G(s) = K \frac{\alpha}{s(s + \alpha)} \qquad (3)$$

when K=$h_{max}$/$u_r$ and α=-m.

Following the procedure given in [8], the system is driven by a stepwise input and their responses are measured as shown in Fig. 5(b). The stepwise input value is the rated motor input, $u_r$. The NCT of the controlled object is then constructed according to the data within the deceleration range. Parameters required for simplified model and the compensator are m, A, $a_{s-res}$, $h_{max}$ and $h_{s-res}$. The simplified dynamic model of the controlled object derived from NCT information is calculated according to the equation (3) are as follow,

$$G_x(s) = 50.7 \frac{1.475}{s(s + 1.475)} \ , \ G_y(s) = 53.1 \frac{3.156}{s(s + 3.156)} \qquad (4)$$

The membership functions of the controlled object for each axis are designed according to the NCT information based on Fig. 5(c).

## IV. SIMULATION RESULTS

Each axis are controlled independently using the same controller structure shown in Fig. 1(a). However, different controller parameters are used according to the individual NCT's. Using a sine wave reference with the X-axis mechanism, and a cosine wave reference to the Y-axis mechanism, the resulting contour is a circle and its circular motion responses is shown in Fig.6.

As shown in Fig 6(b), tracking error to a 1mm amplitude sinusoidal input reference varies within 10 µm except at transient region of X-axis. The root mean square of the x-axis error is 0.0063 mm and 0.0037 mm for y-axis. Radial error to a 1 mm radius circle is within 2 µm, despite its initial radial error related to the transient response of the X-axis. Therefore, initial radius error of the circle contour relatively higher than the rest. However, the root mean square of the radial error is 0.0008 mm.
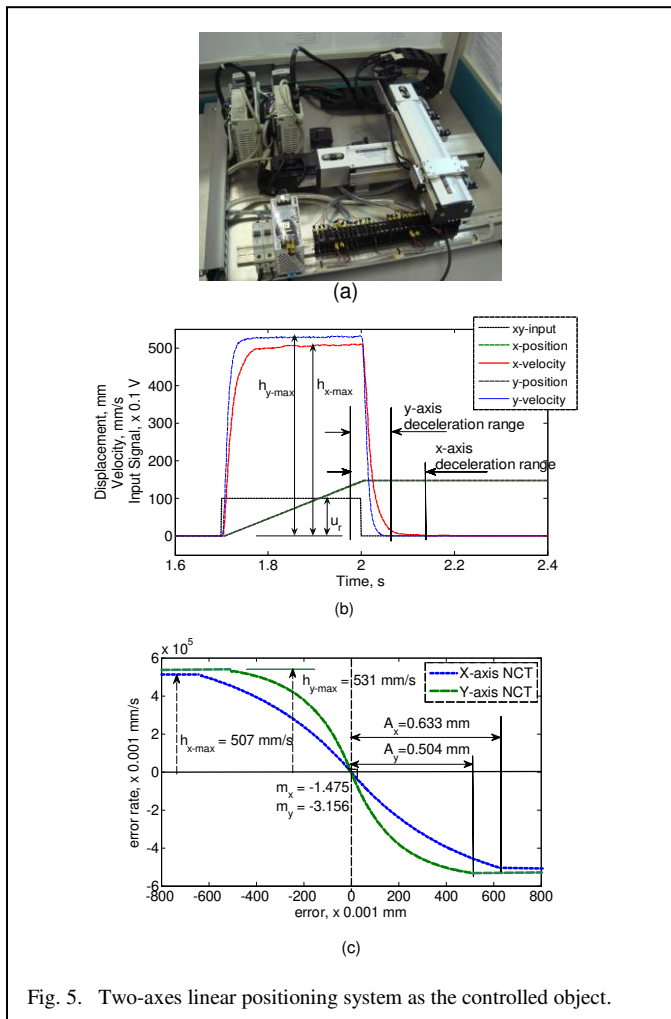


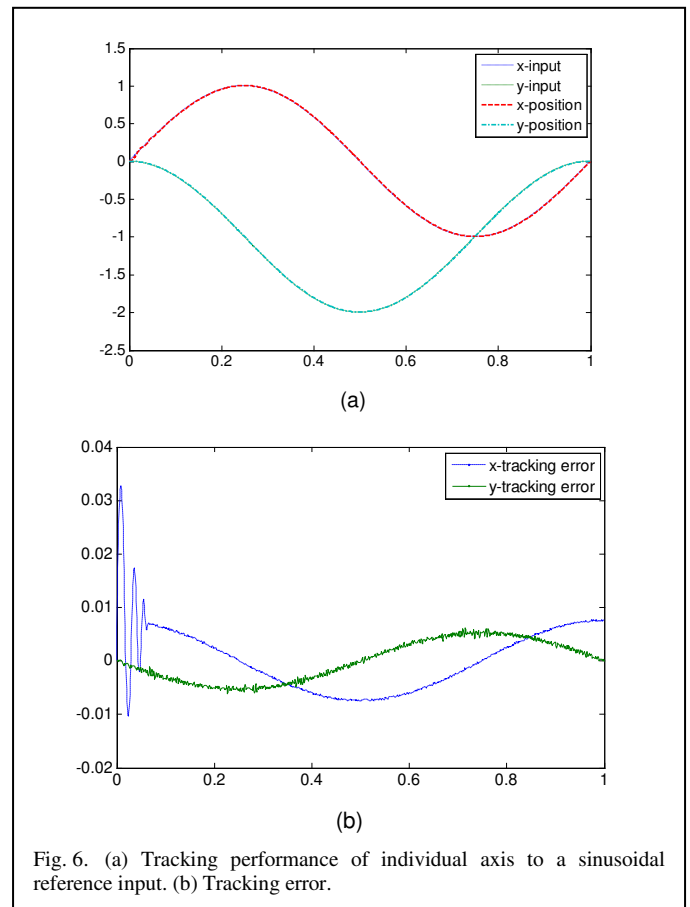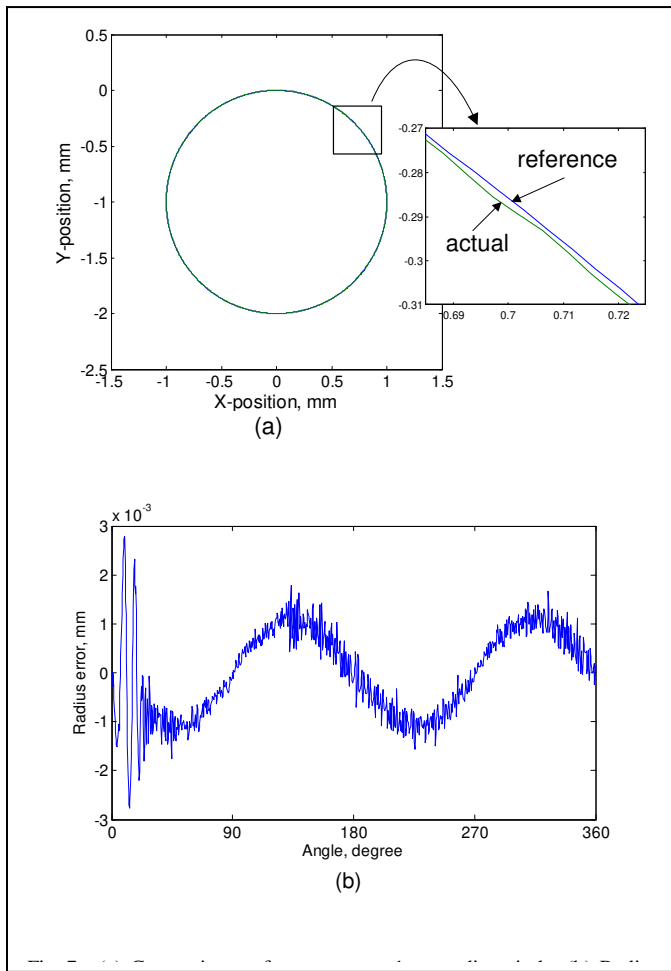Fig. 5. Two-axes linear positioning system as the controlled object.



Fig. 6. (a) Tracking performance of individual axis to a sinusoidal reference input. (b) Tracking error.

Fig. 5. (a) Comparison reference and actual contouring, (b) Radius error

Moreover, the structure and design procedure are kept simple. However, more investigations are required to evaluate its performance compared with another controller and verified through experiment.

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## V. CONCLUSION

This paper evaluates contouring performance of an improved fuzzy-based NCTF controller. The proposed controller was applied to an x-y table to perform continuous circle shape motion. The simulation result indicates that the proposed controller suitable for continuous motion control. The radial error accuracy was within specified sensor resolution.