Effect of MHD on unsteady boundary layer flow past a sphere

[1]Nurul Farahain Mohammad, [2]Abdul Rahman Mohd Kasim, [3]Anati Ali and [4]Sharidan Shafie

[1,2,3,4]Department of Mathematical Sciences, Universiti Teknologi Malaysia Sekudai 81310, Malaysia;
[1]Department of Computational and Theoretical Sciences, International Islamic University Malaysia Kuantan 25200, Malaysia.
Coorresponding Author: nurul.farahain.mohammad@gmail.com

Abstract. Magnetohydrodynamics (MHD) is a field of study which dealing with understanding the dynamic of fluid under the influence of magnetic presence. The effect of MHD in a fluid flow has the potential in controlling the separation of flow, optimizing the heat transfer involved or manipulate the velocity of fluid flow. In this paper, the separation times and flow characteristics of a viscous fluid flow past a sphere are investigated in the presence of magnetic field. In order to solve this problem, the dimensional equations that govern the fluid flow and heat transfer are transformed into dimensionless equations by using appropriate dimensionless variables. The stream functions are introduced to connect and thus having a function that can represent both velocity *u* and *v*. Similarity variables are used to deduce the dimensionless governing equations into a system of nonlinear partial differential equations. This system of equations is solved numerically by using numerical scheme known as the Keller-Box method. The results attained are presented graphically and in tabular form.
Key words: Boundary layer flow, MHD, sphere.

Introduction

The magnetohydrodynamics is the science of the motion of electrically conducting fluids under magnetic fields. This situation is essentially one of mutual interaction between the fluid velocity field and the electromagnetic field; the motion affects the magnetic field and the magnetic field affects the motion. The name of magnetohydrodynamic attempts to convey this relationship (Shercliff, 1965). Accordingly, a considerable amount of research has been accomplished on the effects of electrically conducting fluids such as liquid metals, water mixed with a little acid and others in the presence of transverse magnetic field on the flow and heat transfer characteristics over various geometries (Rahman and Alim, 2009).

But so far there is no investigation has been conducted on unsteady magnetohydrodynamic boundary layer flow past a sphere in viscous fluid. The present paper aims to consider the magnetohydrodynamic unsteady boundary layer flow. In this paper, the governing equations are reduced to nondimensional governing equations. Finally, the nonsimilar governing equations are obtained by using the similarity transformation.

Mathematical Formulation

A boundary layer flow around a sphere of radius a in an unsteady two-dimensional viscous incompressible fluid in the presence of magnetic field is considered. It is assumed that the temperature of the surface of a sphere is T_w. Meanwhile, the acceleration due to gravity g, radial distance from the symmetrical axis to the surface of a sphere r, and velocities (u,v) of (x,y)-axes. The flow starts impulsively at rest with the ambient temperature T_∞ and a uniform stream $(1/2)U_\infty$ at large distance from the sphere and then flows vertically upward. The physical coordinate of the problem is presented in Figure 1.

Under the Boussinesq and boundary layer approximation, the dimensional governing equations are:

Continuity equation:

$$\frac{\partial\left(\bar{r}\bar{u}\right)}{\partial \bar{x}}+\frac{\partial\left(\bar{r}\bar{v}\right)}{\partial \bar{y}}=0, \tag{1}$$

x-momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\mu}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)-\frac{\sigma B_0^2}{\rho}\bar{u}, \tag{2}$$

y-momentum equation:

$$\frac{\partial \bar{v}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial v}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}}+\frac{\mu}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right)-\frac{\sigma B_0^2}{\rho}\bar{v}, \tag{3}$$

Subject to the initial and boundary conditions

$$\begin{aligned} &\bar{t}<0: \quad \bar{u}=\bar{v}=0 \text{ for any } \bar{x}, \bar{y}, \\ &\bar{t}\geq 0: \quad \bar{u}=\bar{v}=0 \text{ at } \bar{y}=0, \\ &\qquad\quad \bar{u}=\bar{u}_e\left(\bar{x}\right) \text{ as } \bar{y}\rightarrow\infty. \end{aligned} \tag{4}$$

where ρ is density of the fluid, μ is viscosity of the fluid, σ is the electrical conduction, B_0 is strength of the magnetic field, $\bar{r}\left(\bar{x}\right)=a\sin\left(\bar{x}/a\right)$ is the radial distance and $\bar{u}_e=\left(3/2\right)U_\infty\sin\left(\bar{x}/a\right)$ is free stream velocity of the fluid.

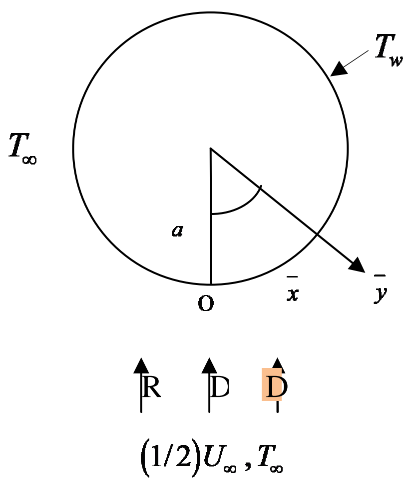


Figure 1 Physical coordinate of the problem

In order to simplify and ease the solving process of the governing equations, some dimensionless variables are introduced as below

$$x=\frac{\bar{x}}{a},\quad y=\mathrm{Re}^{1/2}\frac{\bar{y}}{a},\quad t=\frac{U_\infty\bar{t}}{a},\quad u=\frac{\bar{u}}{U_\infty},\quad v=\mathrm{Re}^{1/2}\frac{\bar{v}}{U_\infty},\quad p=\frac{\bar{p}}{\rho U_\infty^2},\quad r\left(x\right)=\frac{\bar{r}\left(\bar{x}\right)}{a}. \tag{5}$$

Thus, implementing these dimensionless variables in the governing equations produce the nondimensional governing equations as such:
Continuity equation:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial x}=0, \quad (6)$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\partial^2 u}{\partial y^2}-Mu, \quad (7)$$

with the following initial and boundary conditions

$$\begin{aligned} t<0:&\quad u=v=0 \text{ for any } x, y,\\ t\geq 0:&\quad u=v=0 \text{ at } y=0,\\ &\quad u=u_e(x) \text{ as } y\to\infty. \end{aligned} \quad (8)$$

where $\mathrm{Re}=\dfrac{U_\infty a}{\nu}$ is Reynolds number and $M=\dfrac{\sigma B_0^2 a}{\rho U_\infty}$ is magnetic parameter.

In two-dimensional flow, the velocity components in the x- and y- directions at a given points can be expressed by the partial derivative of the stream function ψ using the following relations

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}, \quad \text{and} \quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}. \quad (9)$$

The resulting governing equation in terms of stream function is

$$\frac{1}{r}\frac{\partial^2\psi}{\partial y\partial t}+\frac{1}{r^2}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{1}{r^3}\frac{dr}{dx}\left(\frac{\partial\psi}{\partial y}\right)^2-\frac{1}{r^2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=u_e\frac{du_e}{dx}+\frac{1}{r}\frac{\partial^3\psi}{\partial y^3}+M\left(u_e-\frac{1}{r}\frac{\partial\psi}{\partial y}\right), \quad (10)$$

subjects to the following initial and boundary conditions

$$\begin{aligned} t<0:&\quad \psi=\frac{\partial\psi}{\partial y}=0 \text{ for any } x, y,\\ t\geq 0:&\quad \psi=\frac{\partial\psi}{\partial y}=0 \text{ at } y=0,\\ &\quad \frac{\partial\psi}{\partial y}=u_e(x) \text{ at } y\to\infty. \end{aligned} \quad (11)$$

Immediately after the flow starts, a boundary layer is developed around the sphere with the thickness of $O(\nu t)^{1/2}$. This suggests that in order to obtain a solution for small time $(t\leq t^*)$, the governing equations are transformed by using the following similarity variables

$$\psi=t^{1/2}u_e(x)f(x,\eta,t), \quad \text{and} \quad \eta=y/t^{1/2}. \quad (12)$$

For large time $(t>t^*)$ case, the similarity variables are defined such that

$$\psi=u_e(x)F(x,Y,t), \quad \text{and} \quad \mathrm{Y}=y. \quad (13)$$

Hence, the governing equation become

$$\frac{\partial^3 f}{\partial\eta^3}+\frac{\eta}{2}\frac{\partial^2 f}{\partial\eta^2}+t\frac{du_e}{dx}\left[1-\left(\frac{\partial f}{\partial\eta}\right)^2+f\frac{\partial^2 f}{\partial\eta^2}\right]+Mt\left(1-\frac{\partial f}{\partial\eta}\right)$$
$$=t\frac{\partial^2 f}{\partial\eta\partial t}+tu_e\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial x}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial\eta^2}-\frac{1}{r}\frac{dr}{dx}f\frac{\partial^2 f}{\partial\eta^2}\right), \tag{14}$$

subjects to the following initial and boundary conditions

$$t<0:\ f=\frac{\partial f}{\partial\eta}=0 \ \text{ for any } x,\eta,$$
$$t\geq 0:\ f=\frac{\partial f}{\partial y}=0 \ \text{ at } \eta=0, \tag{15}$$
$$\frac{\partial f}{\partial\eta}=1 \ \text{ at } \eta\to\infty.$$

for small time case. Furthermore, the governing equation for large time case is

$$\frac{\partial^3 F}{\partial Y^3}+\frac{du_e}{dx}\left[1-\left(\frac{\partial F}{\partial Y}\right)^2+F\frac{\partial^2 F}{\partial Y^2}\right]+M\left(1-\frac{\partial F}{\partial Y}\right)$$
$$=\frac{\partial^2 F}{\partial Y\partial t}+u_e\left(\frac{\partial F}{\partial Y}\frac{\partial^2 F}{\partial Y\partial x}-\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial Y^2}-\frac{1}{r}\frac{dr}{dx}F\frac{\partial^2 F}{\partial Y^2}\right) \tag{16}$$

subjects to the following boundary conditions as below

$$F=\frac{\partial F}{\partial Y}=0 \ \text{ at } \ Y=0,$$
$$\frac{\partial F}{\partial Y}=1 \ \text{ as } \ Y\to\infty. \tag{17}$$

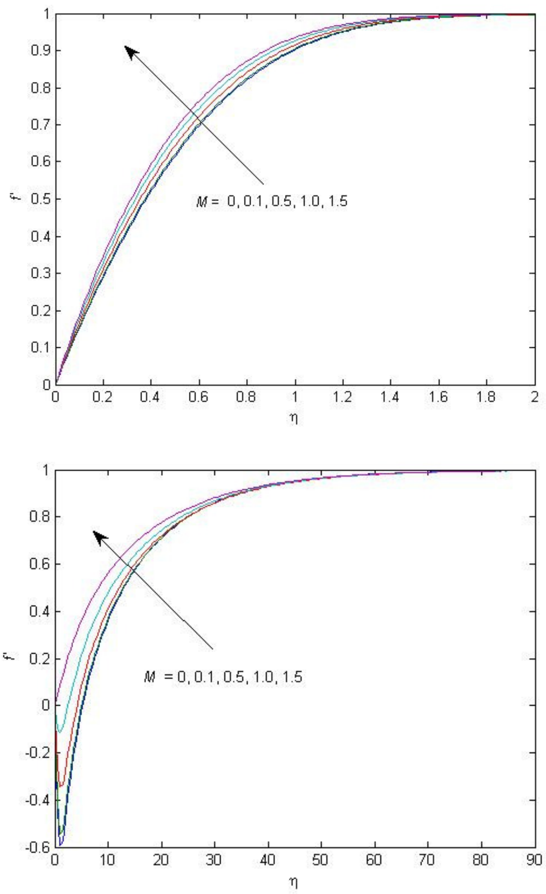
Results and Discussion

The nonsimilar governing equations are solved numerically by using the Keller-Box method in MATLAB programme. The separation times of flow are presented in Table 1. As the values of magnetic parameter are higher, the separation times are getting lower. This phenomenon is also supported by the velocity profiles of the fluid flow in Figure 2. Referring to Figure 2(a), it is clearly seen that there is no separation of flow occurred at the front stagnation point for $0 \leq M \leq 1.5$. While Figure 2(b) shows that there is separation of flow occurred when M = 0.1, 0.5, 1.0 except when M = 1.5. Figure 3 illustrates the variation with x of the skin friction coefficient around a sphere when M = 0 and 1.5. As the time increases, there is skin friction coefficient with negative values in between $100^\circ < x < 180^\circ$ when M = 0. But when M = 1.5, the entire values skin friction coefficient is positive at all points of the surface of a sphere. As the time is getting longer, the values of the skin friction coefficients are getting lower at every point x.

Table 1. The separation times of flow past the surface of a sphere

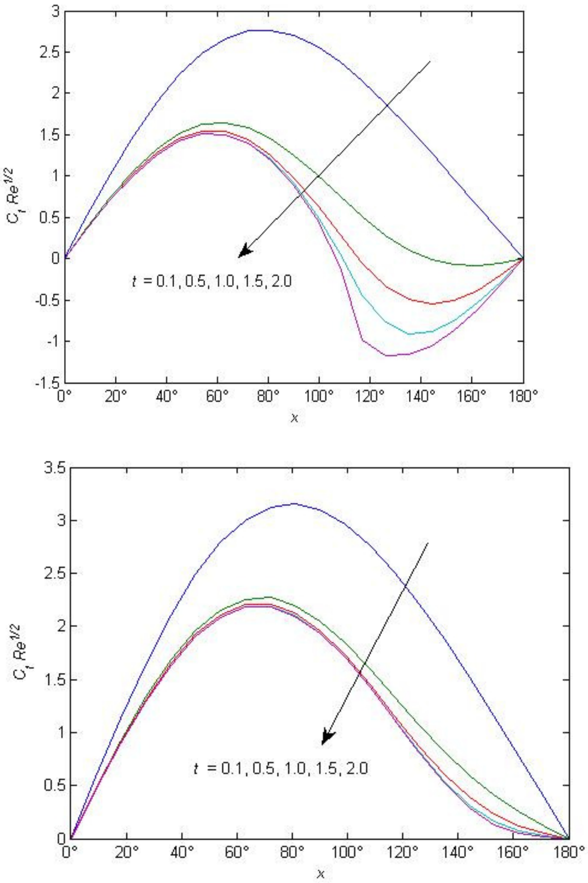
x	*M=0* (Ali,2010)	*M=0* (current)	*M=0.1*	*M=0.5*	*M=1.0*	*M=1.3*
180°	0.3966	0.3960	0.4161	0.5241	0.7963	1.2470
171°	0.4016	0.4010	0.4217	0.5331	0.8186	1.3103
162°	0.4177	0.4170	0.4394	0.5623	0.8940	1.5677
153°	0.4471	0.4463	0.4721	0.6178	1.0627	-
144°	0.4947	0.4937	0.5257	0.7152	1.4428	-

135°	0.5709	0.5694	0.6128	0.8937	-	-
126°	0.6987	0.6960	0.7632	1.2953	-	-
117°	0.9442	0.9372	1.0779	-	-	-
108°	-	1.6751	-	-	-	-



(a) (b)

Figure 2. Velocity profile for various magnetic parameter *M* at (a) front stagnation point and (b) rear stagnation point



(a) (b)

Figure 3. Variation with x of the skin friction coefficient around a sphere when (a) there is no presence of magnetic field (*M*=0) and (b) there is magnetic field (*M*=1.5)

Conclusions

The presence of magnetic field delays the separation flow. When the magnetic parameter $M \geq 1.4$, there is no separation of flow detected at every points of the surface of sphere. This study is also succeed in detecting the separation time of the fluid flow up to 108° which unable to be detected by previous study. There is also no decelaration of flow occurred when the presence of magnetic field is strong enough.

Acknowledgements

The authors would like to acknowledge the Research Management Centre – UTM and MOE for the financial support through votes 4F109, 04H27 and 02H80 for this research.

References

Ali A. 2010. Unsteady Micropolar Boundary Layer Flow and Convective Heat Transfer, PhD Thesis, Universiti Teknologi Malaysia, Malaysia.

Rahman M.M. and Alim M.A. 2009. Numerical study of magnetohydrodynamic free convective heat transfer flow along a vertical flat plate with temperature dependent thermal conductivity. Journal of Naval Architecture & Marine Engineering :16-29.

Shercliff J.A. 1965. A Textbook of Magnetohydrodynamics, Pergamon, Oxford; New York.