# OPTIMIZATION OF WIRELESS PRICING SCHEME

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**ABSTRACT**

The wireless service providers obtain surplus from consumers who applied the service. That pricing strategy is developed by considering the linearity factors, elasticity price, price factors, acceptance factor and unit service price. Previous researches are focussed on the introduction of the models in general. This new approach of the model is by considering the model as the nonlinear programming problem that can be solved optimally using LINGO 13.0. The optimal solution could give information on decision variables and objective function to maximize the revenue for the providers. The several objectives to be achieved by service providers are by setting the increment or decrement of price change due to QoS change and amount of QoS value.

**KEY WORDS**

Optimization, price change, QoS change, amount of QoS value

## 1. Introduction

The pricing scheme has been a critical topic in business. The service providers has the obligation best QoS based on certain services [1, 2]. The research on internet pricing in multi service network in wired networks [3-5], and wired multi QoS network [6, 7] have been discussed. The results mainly inform about the choice of ISP decision to adopt the model by fixing the base price, quality premium and QoS level.

Recently, the development of wireless networks rapidly grows importantly in business life by approaching the network as optimization problem [8]. By using the volume discounts as the nonlinear pricing model the profit of consumers can be achieved. However, due to static condition, the dynamical situation of the models are still in slow progress [9]. Their simulation results show the connection between acceptance factor with the user price elasticity.

Past research [10] focussed on modelling the wireless nonlinear pricing scheme by applying some factors such as the linearity factors, elasticity price and price factors. The idea of modelling the wireless pricing strategy is powerful to be applied in mathematical model.

So, in this paper, we propose the new approach of wireless pricing model originated by [9, 10] by considering the model as the nonlinear programming problem that can be solved optimally using LINGO 13.0. The idea to transform the model into nonlinear programming model is to enable us to identify the connections between the acceptance factor, the price, the revenue, the amount of decrement or increment of QoS change and price change.

## 2. Literature Review

Table I summarized some of those research focusing on the wireless network schemes. Some pricing models do not explicitly describe the availability for QoS differentiation.

Table I

Several Past Research On Internet Pricing

| Pricing Strategy | How it Works |
|---|---|
| Responsive Pricing [11] | Three stages proposed consist of not using feedback and user adaptation, using the closed-loop feedback and one variation of closed loop form. |
| Pricing plan [12] | It Combines the flat rate and usage based pricing. Proposed pricing scheme offers the user a choice of flat rate basic service, which provides access to internet at higher QoS, and ISPs can reduce their peak load. |
| Pricing strategy [1] | Based on economic criteria. They Design proper pricing schemes with quality index yields simple but dynamic formulas'. Possible changes in service pricing and revenue changes can be made |
| Optimal pricing strategy [13] | The schemes are Flat fee, Pure usage based, Two part tariff. Supplier obtains better profit if chooses one pricing scheme and how much it can charge. Two part of analysis homogenous and heterogeneous. |
| Paris Metro Pricing [14, 15] | Different service class will have a different price. The scheme makes use of user partition into classes and move to other class it found same service from other class with lower unit price. |
| Pricing strategy by [16] | Discussion about the measurement of QoS network service performance based on bandwidth, delay and delay jitter, throughput and loss rates. |
| Strategy of pricing proposed by [17] | Pointed out the importance of multiservice networks such as assisting ISPs in spending their allocations, increasing the effectiveness of network usage by giving incentives to customers, to aid well established market view since new services can gain more sustainability. |
| Models for internet pricing proposed by [18] | The utility function of a user can be in the form of probability of packet loss, average packet delay, probability of packet tail, delay of maximum packet and also throughput. |
| Pricing scheme proposed by [19] | Pricing schemes based on QoS levels in different allocations that control congestion and load balance. |

Furthermore, the research on dynamic pricing models and wireless design network is summarized in Table II. The research on this pricing has been begun in last decade and critically improves to fit in dynamical situation in wireless network.

Table II

Some research on dynamic pricing model

| Pricing Strategy | How it Works |
|---|---|
| Pricing for 3G network proposed by [10] | By considering the linearity factor, acceptance factor, elasticity price, the provider able to maximize the price for user and class. |
| Pricing strategy proposed by [20] | By considering the optimal pricing strategy for specific service as function of time. Their proposed model was created then comparing with the existing approaches available. The models focus on continuous models solved heuristically |
| Pricing strategy proposed by [21] | the dynamic pricing scheme proposed by setting up the model as a partial differential equation (PDE) and solving it numerically. The pricing scheme proposed mainly for pricing companies. Their work utilizes the PDE background by utilizing necessary and sufficient condition of Lagrange. So by solving the boundary conditions the pricing scheme involving company debt can be calculated. |
| Social Optimal Pricing by [8] | Pricing strategy that is based on profit maximization of provider. The model is transformed into optimization model. |
| Simulation method for designing network proposed by [22] | Able to examine the schemes that are not reached by network testing and able to improve model and performance. |
| Concept of Dynamic pricing introduced by [23] | The process to fluctuate prices between consumer and provider. In market condition, the re -priced can often occur . |
| Pricing –QoS strategy proposed by [24] | utility function and cost function are proposed, and pricing mechanism is based on QoS service classes. |

## 2. Models

Models used in this framework are adapted from [9, 10] but the approach is the nonlinear programming approach. So the model will consist of the objective function to be maximized subject to sets of constraints. Then, the models are solved using LINGO 13.0 software to obtain the optimal solutions. Based on four cases of the model by considering the increment or decrement of price change due to QoS change and increment or decrement of number of QoS needed we can set up the models required.

Basically, the models attempt to maximize the total price for a connection based on QoS parameter. The total price is the summation between basic price for a connection and the price change due to QoS change. We have i users and j class.

## 3. Result and Discussion

The objective of the research is to obtain the revenue for the provider. The model provided by [10] and then work done by [9] are available. However, we create the models by gathering all information about parameter and variables.

So, the objective function will maximize

$$\sum_{j}^{m}\sum_{i}^{n}\tilde{a}_{ij}(PR_{ij} \pm PQ_{ij})Q_{ij}$$

which means to maximize the revenue that consists of the combination of acceptance factor, the price for a connection with QoS available andthe price change over that QoS and price of unit of service. The objective function has limitation to be satisfied to obtain the revenue which is called the sets of the constraints.

The first constraint states that the price change will depends on the factor of the price, that involves the bandwidth as QoS attribute, the basic price at user i and class j, and also the factor of linearity. Gather all information, we have the sets of the constraints as follow.

$$\mathrm{PQ_{ij}} = (1 \pm \frac{\mathrm{x}}{2000})\mathrm{PB_{ij}Lx}$$

$$\mathrm{PQ_{ij}} = (1 \pm \frac{\mathrm{x}}{350})\mathrm{PB_{ij}Lx}$$

where$PB_{ij}$ is the basic price for a connection for user i and the class j and Lx is the linearity factor. The QoS attributes used are bandwidth and end to end delay. Then, $a_{ij}$which defines the linear price factor in user *i* and class j, the linear factor $(\mathrm{e} - \mathrm{e}^{-\mathrm{Bx}})$ and the traffic load $t_l$. So,

$$\mathrm{PB_{ij}} = \mathrm{a_{ij}}(\mathrm{e} - \mathrm{e}^{-\mathrm{Bx}})\mathrm{t_l}/100$$

Lx is a linearity factor that depends on the linearity parameters of a and $(\mathrm{e} - \mathrm{e}^{-\mathrm{Bx}})$. Then

$$\mathrm{Lx} = \mathrm{a}(\mathrm{e} - \mathrm{e}^{-\mathrm{Bx}})$$

With x is assumed between 0 and 1.

The traffic load will be determined by setting the range for the traffic load is between the prescribed value arranged by the providers.

The linear price factor $a_{ij}$ is set up between prescribed values determined by the provider, say *f* and *g*. So,

$$\mathrm{f} \le \mathrm{a_{ij}} \le \mathrm{g}$$

The range of allowed traffic load $\mathrm{t_l}$ is also determined by the providers, say h and k. Then,

$$\mathrm{h} \le \mathrm{t_l} \le \mathrm{k}$$

For *x* as the amount of increment of decrement in QoS value, we range between 0 and 1 implying 0 is in best effort service case while 1 means in perfect service case. B is arranged between 0.8 and 1.07 since in this range, the best network quality occurs [10].

$$0 \le \mathrm{x} \le 1$$

$$0.8 \le \mathrm{B} \le 1.07$$

For parameter value $PR_{ij}$, the provider arranges the value to have a connection. It also happens in a as the linearity parameters that keep the ratio of the price between floor and ceiling of QoS value is not really high.

Next step, for a model described above, the optimal solution for 4 cases involving decrement or increment of price change due to change of QoS and decrement or increment of QoS value is conducted by using LINGO 13.0. Table II and III summarize the solver status for all cases and the decision variables, respectively.

Table III

Solver Status Of Nonlinear Programming Model Of Wireless Pricing Scheme

| variables | *PQij*increase *x* increase | *PQij*increase x decrease | PQij decrease x increase | PQij decrease x decrease |
|---|---|---|---|---|
| Model Class | NLP | NLP | NLP | NLP |
| State | Local Optimal | Local Optimal | Local Optimal | Local Optimal |
| Objective | 435.443 | 435.443 | 42.52 | 43.58 |
| Infeasibility | 5.9x10-11 | 2.4x10-12 | 3.1x10-8 | 1.07x10-13 |
| Iterations | 23 | 22 | 22 | 22 |
| GMU | 30 | 30 | 30 | 30 |
| ER | 1s | 0 | 1s | 0s |

In Table III, model class for each class I defined as nonlinear programming, having local optimal state. The best objective value to maximize the price for each user is achieved when $PQ_{ij}$ increases with decrease of x. Iterations involve in the highest objective value is the lower or the same value with other case.

Next, in Table IV, the decision variables for 2 users and 2 classes are presented. The price change due to QoS change for each case appears to have different value if we increase or decrease the condition of the change. The value of linearity parameter B, in three cases is the ceiling of the requirement set up for B. The value of the unit of service price is the same value for all cases. The traffic load value is the floor of the predetermined range while the linearity factor has the same value for all cases. It is shown in the Table IV that by setting up the increment of the price change due to QoS change and decrement the amount of QoS change, the providers gain best revenue.

Table IV

Decision Variables Of Nonlinear Programming Model Of Wireless Pricing Scheme

| variables | *PQij*increase *x* increase | *PQij*increase *x* decrease | *PQij*decrease *x* increase | *PQij* decrease *x* decrease |
|---|---|---|---|---|
| $PQ_{11}$ | 4.42 | 4.42 | 0.07 | 0.07 |
| $PQ_{12}$ | 4.13 | 4.13 | 0.11 | 0.08 |
| $PQ_{21}$ | 3.83 | 3.83 | 0.11 | 0.1 |
| $PQ_{22}$ | 3.54 | 3.54 | 0.11 | 0.11 |
| $x$ | 0 | 0 | $0.4x10^{-6}$ | 0 |
| $PB_{11}$ | 2.57 | 2.57 | 0.04 | 0.04 |
| $PB_{12}$ | 2.4 | 2.4 | 0.06 | 0.05 |
| $PB_{21}$ | 2.23 | 2.23 | 0.06 | 0.06 |
| $PB_{22}$ | 2.06 | 2.06 | 0.06 | 0.07 |
| $a_{11}$ | 0.15 | 0.15 | 0.05 | 0.05 |
| $a_{12}$ | 0.14 | 0.14 | 0.05 | 0.06 |
| $a_{21}$ | 0.13 | 0.13 | 0.08 | 0.07 |
| $a_{22}$ | 0.12 | 0.12 | 0.08 | 0.08 |
| $B$ | 1.07 | 1.07 | 0.8 | 1.07 |
| $Q_{11}$ | 10 | 10 | 10 | 10 |
| $Q_{12}$ | 10 | 10 | 10 | 10 |
| $Q_{21}$ | 10 | 10 | 10 | 10 |
| $Q_{22}$ | 10 | 10 | 10 | 10 |
| $\tilde{a}_{11}$ | 0.15 | 0.15 | 0.15 | 0.15 |
| $\tilde{a}_{12}$ | 0.15 | 0.15 | 0.15 | 0.15 |
| $\tilde{a}_{21}$ | 0.15 | 0.15 | 0.15 | 0.15 |
| $\tilde{a}_{22}$ | 0.15 | 0.15 | 0.15 | 0.15 |
| $t_l$ | 50 | 50 | 50 | 50 |
| $Lx$ | 1.7 | 1.7 | 1.7 | 1.7 |

## 4. Conclusion

The goal to maximum price is achieved when the provider set the increment of price change due to QoS change and the decrement of amount of QoS value. The QoS attribute used is bandwidth and end to end delay. The linearity parameter set up for most cases is obtained in ceiling value. Linear price factor ranges between the prescribed values especially cases when we increase the price change due to QoS change and increase the amount of QoS values.

## Acknowledgement(s)

Theresearchleading to this paper was financially supported by Ministry of Higher Education Malaysia for support through Fundamental Research Grant Scheme (FRGS) 2014.

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