# An optimization technique to prepare nurse schedule for a monthly time horizon

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**Abstract.** Nurse scheduling problem is one of the most difficult scheduling problems to solve since its solution space is large and it expects to comply many constraints. There is no standard model or a method of solution for nurse scheduling. The main objective of this study is to search for a scientific method to prepare a monthly working schedule for a group of nursing officers employed in a hospital. We propose an optimization method to prepare an optimal schedule. Initially, we develop an optimization model by formulating the objective and the constraints of the problem. The optimization model that we are interested in is a 0-1 Integer Linear Programming problem. We apply the Branch-and-Bound technique to solve the problem using the optimization software package LINGO. Finally, the solution to the optimization problem is formulated to a regular nurse schedule. The methodology is illustrated by preparing a monthly schedule for a private hospital in Sri Lanka.
**Keywords:** Nurse scheduling, Mathematical Programming, Meta-Heuristic Methods, Constraint Satisfaction Techniques, Column generation, Set Covering–Type models, Knowledge Based Techniques, Branch-and-Bound technique.

## Introduction

Personal scheduling means assigning employees into shifts over a specific scheduling period in an optimal manner considering all necessary factors. Nurse scheduling problem is considered as NP-hard combinatorial problem (Karp et al. 1972) in the field of personal scheduling. Any hospital management put a lot of effort to prepare schedule for their nursing officers while being perceived to be fair by the officers. This task is challenging due to different nurse requirements on different days and shifts, and also due to hospitals being in continuous operation unlike most of the other institutions. Therefore, there is a need for an efficient approach to schedule nurses in order to save the time and resources in the scheduling process and to conduct hospital operations smoothly.

In the past, a wide variety of methodologies and models have been developed to deal with different problem instances. Introduction to many of these methods can be found in (E.K. Burke et al. 2005). A number of survey papers (Burke et al. 2004; Cheang et al. 2003) give an overview of the area. The available techniques to schedule nurses can roughly be divided into two main categories: exact algorithms and metaheuristics. Mathematical programming is the traditional exact method (Bard et al. 2005; Beaumont et al. 1997), which guarantees to find an optimal solution on nurse scheduling for every instance of a problem. However, computational difficulties exist with this approach due to the enormous size of the search spaces that are generated. To reduce complexity, some researchers have restricted the problem dimensions and developed simplified models. However, this leads to solutions that are not applicable to real situations.

Since 1990's Metaheuristic approaches (Smith et al. 1979; Blau et al. 1985) were investigated with some success. Genetic and Memetic algorithms form an important class of metaheuristics that have been extensively applied in nurse scheduling (Aickelin et al. 2000, 2004; Burke et al. 2001). A number of attempts have also been made by using other metaheuristics, such as simulated annealing ( Brusco et al.1993), tabu search (Burke et al.1999), variable neighborhood search (Burke et al. 2004,2008,2010) and estimation of distribution algorithms (Aickelin et al.2007). However, the major drawback of these metaheuristics is that in most of the cases they cannot produce optimal solutions nor reduce the search space. Also, they usually do not have well defined stopping criteria. Moreover, most nurse scheduling problems are highly constrained problems so that the feasible regions of their solution space are disconnected. Metaheuristics generally have difficulty in dealing with this situation. In the field of nurse scheduling, some decomposition techniques have been investigated over recent passed years. Aickelin and Dowsland (Aickelin et al.2004) developed a genetic algorithm with an indirect representation. Different heuristic decoders (i.e. decomposers) were employed to construct the schedule, taking care of coverage and nurses' preferences from different aspects. Ikegami (Ikegami et al. 2003) grouped the constraints into shift constraints and nurse constraints. Based on this, the problems were decomposed into sub-problems and solved repeatedly by tabu search.

In this paper a mechanism based on 0-1 integer programming is proposed to prepare a roster for a group of nurses attached to a private hospital in Sri Lanka for a period of one month. This proposed method is based on Branch-and-Bound technique and it is therefore less complex compared to the existing methods in the scheduling field. Further, this approach not just provides an idea to prepare a roster but includes some additional features which are not so easy to include when preparing a roster manually. These features are elaborated in the proposed mathematical model.

In the research paper (Paramadevan et al. 2013) minimum number of working days and minimum number of night shifts were not considered. Therefore, there were variations in the total number of working shifts in the time period of one week. In fact some nurses did not get even a single working shift in that week due to their high penalty values. However, it provides an optimal schedule satisfying nurses' preference as well as conditions stipulated by the hospital administration. The proposed method presented in that paper, the schedule must be prepared weekly which is an extra burden for the hospital administration.

In this paper, two constraints are included to overcome these drawbacks and the model is enhanced further to monthly schedule and the demand is increased to four. This model provides a schedule to the nurses well in advance which is lacking in the model presented in (Paramadevan et al. 2013). Also, by adjusting certain parameters in the proposed model, the scheduling period can be extended even to quarter of a year at the expense of computational time.

The rest of the paper is organized as follows. Following section describes materials and methods and the subsequent section present results and discussion. Finally, we conclude the paper with our conclusion.

**Materials and Methods**

Prior to the optimization model, we introduce the notations and meanings of technical terms that are being used. Next we present the optimization model to solve the nurse scheduling problem.

1. Let $N = \{1,2,.......,\hat{n}\}$ be the set of nurses. For any $n \in N, Y_n, Z_n, y_n$ and $z_n$ are the maximum number of day shifts, maximum number of night shifts, minimum number of day shifts and minimum number of night shifts to be assigned for the nurse $n$ within the scheduling period respectively.
2. Let $T = \{1(early), 2(day), 3(late), 4(night)\}$ be the set of shift types of equivalent length of 6 hours each day.
3. Let $D = \{1,2,.......,\hat{d}\}$ be the days in the scheduling period.
4. Let $R_{d\_t}$ be demand of number of nurses on day $d \in D$ of shift type $t \in T$.
5. Let $k_1$ be the maximum number of consecutive day shifts.
6. Let $k_2$ be the maximum number of consecutive night shifts.
7. Let $p_{n\_d\_t}$ be a penalty occurring if the nurse $n \in N$ is scheduled to work at shift type $t \in T$ on day $d \in D$.

Our task is to assign nurses for different shift types according to the demand on each day in the planning time horizon. It should satisfy the following standard requirements which are common to most of the nurse scheduling problems.

I. Demand at a shift type on a particular day should be satisfied.
II. A nurse should be assigned for at most one shift a day.
III. At most maximum number of day shifts should be assigned for a nurse.
IV. At least minimum number of day shifts should be assigned for a nurse.
V. At most maximum number of night shifts should be assigned for a nurse.
VI. At least minimum number of night shifts should be assigned for a nurse.

In addition, following special requirements are requested by the nursing officers:

VII. No night shifts between two non-night shifts.
VIII. Shift 1 or Shift 2 should not be assigned after a night shift.
IX. Maximum number of consecutive day shifts is 5.
X. Maximum number of consecutive night shifts is 3.

Next we introduce the decision variables.
Let

$$x_{n\_d\_t} = \begin{cases} 1, \text{if nurse n is assigned for the shift type t on day } d \\ 0, \text{other wise.} \end{cases}$$

Introducing these decision variables, the optimization model which incorporates both common and special requirements can be presented as:

Minimize $\sum_{n\in N}\sum_{d\in D}\sum_{t\in T} p_{n\_d\_t}\, x_{n\_d\_t}$

Subject to

$$\sum_{n\in N} x_{n\_d\_t} = r_{d\_t}, \ \forall d\in D, t\in T$$

$$\sum_{t\in T} x_{n\_d\_t} \le 1, \quad \forall n\in N, d\in D$$

$$\sum_{d\in D}\sum_{t\in T} x_{n\_d\_t} \le Y_n, \quad \forall n\in N$$

$$\sum_{d\in D}\sum_{t\in T} x_{n\_d\_t} \ge y_n, \quad \forall n\in N$$

$$\sum_{d\in D} x_{n\_d\_4} \le Z_n, \quad \forall n\in N$$

$$\sum_{d\in D} x_{n\_d\_4} \ge z_n, \quad \forall n\in N$$

$$x_{n\_d-1\_4} - x_{n\_d\_4} + x_{n\_d+1\_4} \ge 0, \ \forall n\in N, d\in \{2,3,......,\hat{d}-1\}$$

$$x_{n\_d-1\_4} + x_{n\_d\_1} + x_{n\_d\_2} \le 1, \ \forall n\in N, d\in D$$

$$\sum_{d=l}^{l+k_1} x_{n\_d\_t} \le k_1, \quad \forall n\in N, l\in \{1,2,.....,\hat{d}-k_1\}$$

$$\sum_{d=l}^{l+k_2} x_{n\_d\_4} \le k_2, \quad \forall n\in N, l\in \{1,2,.....,\hat{d}-k_2\}$$

The mathematical model described above is to prepare a monthly schedule for 24 nurses at a private hospital in Sri Lanka. These schedules must respect working contracts and meet the demand for a given number of nurses on each shift type, while being perceived to be fair by the nurses themselves. The objective function considered in the optimization model is a cost function, where cost is interpreted as penalty and penalty is defined based on the desirability of a nurse to work at a shift type on a day. Therefore, our attempt is to minimize the penalty subject to the given constraints. These penalties are appeared as coefficients in the objective function. Hospital administration requires 4 nurses at each shift so the demand $R_{d\_t}$ is uniform and is equal to 4 for all shifts throughout the time horizon. Also, maximum number of day shifts, minimum number of day shifts, maximum number of night shifts and minimum number of night shifts for a nurse are uniform and are assumed to be 25, 20, 10 and 5 respectively. That is, $Y_n = 25, y_n = 20, Z_n = 10$ and $z_n = 5$ for each nurse. Further, both the maximum number of consecutive day shifts, $k_1$ and maximum number of consecutive night shifts, $k_2$ within the scheduling period are required to be 5 and 3 respectively.

**Results and Discussion**

This proposed model is applied to a private hospital with 24 nurses. By using the information obtained from the hospital administration as well as nursing officers, a 0-1 integer linear programming model is formulated, which consists of 2976, 0-1 binary decision variables and 3677 constraints. Computations are carried on a SAMSUNG, Intel(R) Core(TM) i3 processor and 4 GB RAM. The result obtained by solving the 0-1 Integer Linear Programming problem is presented in the table given below. Here, numbers 1,2,3 and 4 represent the shift type for the corresponding days:

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 2 | 2 | ■ | 4 | ■ | ■ | 3 | 4 | 4 | ■ | ■ | 3 | 3 | 3 | 1 | 1 | 4 | ■ | 1 | 2 | 2 | ■ | ■ |
| **2** | 4 | ■ | ■ | 2 | 3 | ■ | ■ | 1 | 4 | 4 | ■ | 1 | 2 | 3 | 1 | 1 | ■ | 3 | 3 | 2 | ■ | ■ | 4 | 2 |
| **3** | 4 | 2 | ■ | 1 | 4 | 3 | 3 | 3 | ■ | ■ | ■ | ■ | ■ | 2 | 4 | ■ | ■ | 1 | 2 | 3 | 1 | 2 | 4 | 1 |
| **4** | ■ | 4 | 3 | 3 | 4 | 1 | 2 | 1 | 2 | ■ | ■ | 2 | ■ | ■ | 4 | 2 | 3 | ■ | 1 | 1 | 4 | ■ | ■ | 3 |
| **5** | 2 | 4 | 1 | ■ | ■ | 3 | 2 | ■ | 4 | ■ | ■ | 3 | ■ | 1 | 4 | 2 | 3 | 2 | 3 | ■ | 4 | ■ | 1 | 1 |
| **6** | 2 | 4 | 3 | ■ | 2 | ■ | 2 | ■ | 4 | 3 | ■ | 1 | 4 | 3 | ■ | 1 | 2 | 1 | 1 | ■ | 4 | ■ | 3 | ■ |
| **7** | 3 | ■ | 2 | | 1 | 2 | 1 | 2 | 3 | ■ | ■ | 1 | 4 | 4 | ■ | 4 | 4 | 2 | ■ | 3 | 3 | ■ | 1 | ■ |
| **8** | ■ | 2 | 2 | 3 | 2 | 3 | ■ | 3 | ■ | 1 | 1 | ■ | ■ | 4 | ■ | 4 | 4 | 2 | 3 | 4 | ■ | 1 | ■ | 1 |
| **9** | 3 | 1 | ■ | 2 | 2 | 3 | 1 | ■ | 1 | 4 | ■ | ■ | ■ | 4 | 1 | ■ | ■ | 2 | 4 | 4 | 3 | 3 | 2 | ■ |
| **10** | 2 | 2 | ■ | 1 | ■ | ■ | ■ | 3 | ■ | 4 | 2 | 3 | 3 | ■ | 1 | ■ | 1 | ■ | 4 | 4 | 2 | 1 | 4 | 3 |
| **11** | 3 | ■ | 1 | 2 | ■ | ■ | 2 | 2 | 3 | ■ | 1 | 4 | ■ | 1 | 2 | 4 | 1 | 3 | 4 | ■ | ■ | ■ | 4 | 3 |
| **12** | ■ | 3 | ■ | 3 | 1 | 4 | ■ | ■ | ■ | 2 | 3 | 4 | ■ | 1 | 4 | 4 | 2 | 2 | ■ | 1 | 3 | 1 | ■ | ■ |
| **13** | 2 | 1 | 1 | ■ | 3 | 4 | ■ | 2 | 4 | 3 | 3 | | 1 | 1 | 4 | ■ | 2 | ■ | ■ | 3 | 4 | ■ | ■ | 2 |
| **14** | ■ | 3 | 3 | ■ | ■ | 3 | ■ | 3 | 4 | 1 | 1 | 2 | 2 | 1 | ■ | 2 | ■ | 1 | ■ | ■ | 4 | 4 | 4 | 2 |
| **15** | 2 | ■ | 1 | 4 | 2 | ■ | 4 | 3 | 3 | ■ | ■ | 2 | 1 | 3 | 1 | ■ | 1 | 2 | 3 | ■ | ■ | 4 | 4 | ■ |
| **16** | 2 | 1 | ■ | 4 | 3 | 2 | 4 | ■ | ■ | ■ | 3 | ■ | ■ | ■ | 1 | 4 | 1 | 1 | 3 | 2 | 2 | 4 | ■ | 3 |
| **17** | ■ | 4 | 4 | | 1 | 2 | 4 | ■ | 2 | 1 | 1 | 2 | 1 | ■ | ■ | 4 | ■ | ■ | 3 | 3 | 3 | ■ | 2 | 3 |
| **18** | ■ | 4 | 4 | 4 | 2 | 2 | 3 | ■ | 2 | ■ | 3 | 1 | 3 | ■ | ■ | ■ | 1 | 1 | ■ | 4 | 1 | ■ | 2 | 3 |
| **19** | ■ | 3 | 4 | 4 | 3 | 3 | ■ | 4 | ■ | 1 | 1 | 2 | ■ | 1 | 2 | 3 | ■ | 1 | ■ | 4 | 2 | 2 | ■ | ■ |
| **20** | 2 | ■ | ■ | 4 | ■ | ■ | 1 | 4 | 3 | ■ | 2 | 3 | 4 | ■ | 1 | 3 | 2 | 1 | ■ | ■ | 1 | 2 | 3 | 4 |
| **21** | 2 | ■ | 3 | ■ | 3 | 2 | 2 | ■ | 1 | 3 | ■ | ■ | 4 | 4 | 2 | ■ | 4 | 2 | 1 | 1 | ■ | ■ | 3 | 4 |
| **22** | ■ | ■ | 4 | 2 | 3 | 2 | ■ | ■ | 3 | 3 | 2 | 1 | 4 | 4 | ■ | ■ | 4 | 1 | ■ | 1 | 2 | 1 | 2 | ■ |
| **23** | 2 | 2 | 4 | 3 | ■ | ■ | ■ | 1 | ■ | 4 | 1 | 2 | ■ | 3 | 3 | ■ | 4 | ■ | 2 | 2 | 3 | 4 | 1 | ■ |
| **24** | 3 | ■ | 3 | ■ | 1 | 3 | ■ | 2 | 2 | 4 | 4 | ■ | 1 | ■ | ■ | 3 | ■ | 4 | 1 | 1 | ■ | 4 | 2 | 2 |
| **25** | 1 | ■ | ■ | 2 | 4 | 4 | 3 | 2 | ■ | ■ | 4 | 3 | 1 | 1 | ■ | ■ | 1 | 4 | 2 | 3 | ■ | 3 | ■ | 2 |
| **26** | ■ | 3 | 2 | 2 | 4 | 4 | ■ | 4 | 2 | 1 | 3 | 1 | 2 | ■ | ■ | 3 | 1 | ■ | ■ | ■ | 1 | ■ | 3 | 4 |
| **27** | ■ | 2 | 3 | ■ | ■ | 4 | 1 | 4 | 3 | 3 | ■ | 4 | 3 | ■ | 2 | 2 | 1 | ■ | 1 | ■ | ■ | 2 | 1 | 4 |
| **28** | 4 | ■ | ■ | 1 | ■ | ■ | 1 | ■ | 1 | 1 | 3 | 4 | ■ | 3 | 2 | ■ | ■ | 2 | 4 | 3 | 2 | 3 | 2 | 4 |
| **29** | 4 | 3 | 2 | ■ | 2 | 2 | 1 | ■ | ■ | 3 | 4 | 4 | 1 | ■ | ■ | 1 | ■ | 2 | 4 | 1 | 3 | 3 | ■ | ■ |
| **30** | 4 | 1 | ■ | 1 | 1 | ■ | 4 | 3 | ■ | ■ | 4 | ■ | 2 | 2 | 2 | 2 | ■ | 4 | ■ | ■ | 3 | 3 | 3 | 1 |
| **31** | ■ | ■ | ■ | 1 | ■ | ■ | 4 | 4 | 2 | 3 | 4 | ■ | 2 | 1 | 3 | 2 | 3 | 4 | 3 | ■ | ■ | 3 | 1 | 2 |

First row represents nurse number and the first column represents day number. Black slots in each column indicate free shift for the corresponding nurse. The corresponding model can be obtained by simply changing the value of the parameter called time horizon in the C++ code in case of 30 or less days per month.

**Conclusion**

We have developed an optimization model according to the given conditions for nurse scheduling. We have also proposed a method of solution to solve the model. This method will guide to prepare a nurse schedule more efficiently, accurately and quickly for any hospital which satisfies the nurses' preferences and regulations provided by the hospital administrators. This model support human schedulers when schedule nurses for long time horizon. This aspect may help in various scheduling problems. In our model, all nurses are considered as same grade. But there are hospitals, where nurses are categorized into different grades according to their qualifications/experience. In fact proposed model has to be modified to suite to situations where nurses are categorized into different grades. It is worth to emphasize that this model support to prepare a long term schedule for the nurses. In fact, it is always preferable to have a long term scheduling period, because nurses can plan their personal work if they know their roster well in advance. Also, preparing a monthly schedule reduces the work load of the hospital administration significantly.

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