# Investigating Performance Zero-Forcing of Source Weighting Matrix in MIMO Relay Communication

**Apriana Toding[1], Rismawaty Arungla'bi[2]**
**Dept. Electrical Engineering[1,2]**
**Universitas Kristen Indonesia Paulus, Makassar, Sul-Sel 90245, Indonesia**
**aprianatoding@ukipaulus.ac.id[1], rismawaty@ukipaulus.ac.id[2]**

**ABSTRACT**

In this paper, we study the zero forcing (ZF) algorithm for a two hop multiple-input multiple-output (MIMO) relay communication system consisting of one source node, one relay node, and one destination node. We consider amplify-and-forward (AF) strategy at relay node and destination node, respectively. We propose Pre-ZF equalization on the transmitter side. We have evaluated the proposed system by comparing with existing algorithms. The proposed Pre-ZF equalization algorithm has a significant improvement in the system bit-error-rate (BER) performance.

**Key words**: multiple-input multiple-output (MIMO), MIMO relay networks, relay system, beamforming, zero forcing.

## 1. Introduction

In order to provide a reliable wireless transmission, one needs to compensate for the effects of signal fading due to multi-path propagation and strong shadowing. One way to address these issues is to transmit the signal through one or more relays [1]-[6], which can be accomplished via wireless network consisting of geographically separeated nodes. And then the basic motivation behind the use of cooperative communications lies in exploitation of spatial diversity provided by the network nodes [7] and [8], as well as the efficient use of power resources [9]-[14] which can be achieved by a scheme that simply receives and forwards a given information, yet designed under certain optimality criterion.

Relay schemes can be broadly categorized into three general groups: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). in the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [3]-[6]. In the DF scheme, the relay nodes first decode the received signals and then forward the re-encoded signals toward the destination node [7]. In the CF method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [16]. In this paper we consider the AF strategy which is easier to implement compared with the other two approaches.

When nodes in the relay system are installed with multiple antennas, we call such system multiple-input multiple-output (MIMO) relay communication system. Recently, MIMO relay communication system have attracted much research interest and provided significant improvement in terms of both spatial efficiency and link reliability [4], [6], and [13]-[16]. In this paper, we investigated the performance of zero forcing (ZF) relay communication system consisting of one source node, relay node, and one destination node in terms of bit error rate (BER). Note that the ZF algorithm has already been studied with single-hop MIMO [17] and MIMO relay channel [18]. And then, in [15], pre-ZF has been investigated for single-hop MIMO channel and two-hop MIMO relay channel. We have evaluated the proposed system by comparing with existing algorithm. The proposed Pre-ZF equalization algorithm has a significant improvement in the system bit-error-rate (BER) performance.

The rest of the paper is organized as follows: the system model is described in Section II; In section III we study the weight matrix on the transmitter side in a MIMO relay system; Section IV shows the simulation results which justify the performance gain with relay Pre-ZF algorithm and the conclusion is given in Section V.

## 2. System Model

### 2.1 General Assumptions

Fig. 1 illustrates a two-hop MIMO relay communication system consisting of one source node, $K$ parallel relay nodes, and one destination node. We assume that the source and destination nodes have $N_s$ and $N_d$ antennas, respectively, and each relay nodes has $N_r$ antennas.

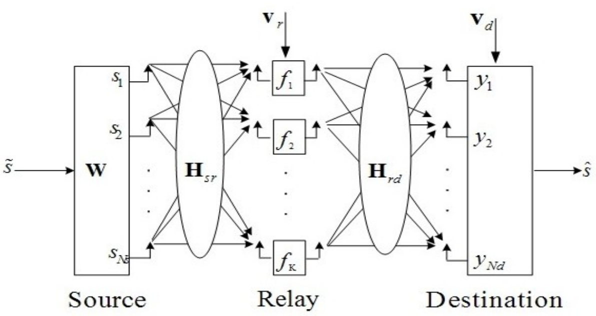


Fig. 1. System model.

The generalization to the system with different number of antennas at each relay node is straightforward. To efficiently exploit the system hardware, each relay node uses the same antennas to transmit and receive signals. Due to its merit of simplicity, we consider the amplify-and-forward scheme at each relay.

### 2.2 System Model

Throughout the paper, we make a practical assumption that the source node and the relay nodes have their own transmit power constraints, i.e., $P_s$ and $P_r$. All channels are assumed to be slow, frequency flat Rayleigh fading model which the coefficients are independent and identically distributed (i.i.d), circularly symmetric complex Gaussian random variables with zero mean and unit variance. The communication process between the source and destination nodes is completed in tow time slots. In the first time slot, the $N_s$ x 1 modulated symbol vector ŝ is linearly precoded as

$$\boldsymbol{s} = \boldsymbol{W}\hat{\mathbf{s}} \qquad \textbf{(1)}$$

Where $\mathbf{W}$ is an $N_s \, x \, N_s$ source weight matrix. $\hat{S}$ is the $N_s$ x 1 transmit data vector with $\mathbf{E}[\hat{s}\,\hat{s}^H] = I_{Ns}$.

The precoded vector S is transmitted to the relay nodes. The received signal at the $i^{th}$ relay node can be written as

$$\boldsymbol{y}_{r,i} = \boldsymbol{H}_{sr,i}\boldsymbol{s} + \boldsymbol{v}_{r,i} \qquad \textbf{(2)}$$

$i = 1, \ldots, K$

where $\boldsymbol{H}_{sr,i}$ is the $N_r \, x \, N_s$ MIMO channel matrix between the source and the $i^{th}$ relay node, $\boldsymbol{y}_{r,i}$ and $\boldsymbol{v}_{r,i}$ are the received signal and the additive Gaussian noise vectors at the $i^{th}$ relay node, respectively.

In the second time slot, the source node is silent, while each relay node transmits the amplified signal vector to the destination node as

$$\mathbf{x}_{r,i} = \mathbf{f}_i \mathbf{y}_{r,i} \qquad \textbf{(3)}$$

$i = 1, \ldots, K$

where $\mathbf{f}_i$ is the $N_r \, x \, Nr$ amplifying matrix at the $i^{th}$ relay node. Thus the received signal vector at the destination node can be written as

$$y_d = \sum_{i=1}^{K} H_{rd,i}\, x_{r,i} + v_d \qquad \textbf{(4)}$$

Where $H_{rd,i}$ is the $Nd \, x \, Nr$ MIMO channer matrix between the $i^{th}$ relay and the destination node, $y_d$ and $v_d$ are the received signal and the additive Gaussian noise vectors at the destination node, respectively.

Substituting (1)-(3) into (4), we have

$$y_d = \sum_{i=1}^{K} \boldsymbol{H}_{rd,i}\boldsymbol{f}_i\boldsymbol{H}_{sr,i}\boldsymbol{W}_{\hat{s}} + \boldsymbol{H}_{rd,i}\boldsymbol{f}_i\boldsymbol{v}_{r,i} + \boldsymbol{v}_d \qquad (5)$$

$$= \boldsymbol{H}_{rd}\boldsymbol{F}\boldsymbol{H}_{sr}\boldsymbol{W}_{\hat{s}} + \boldsymbol{H}_{rd}\boldsymbol{F}\boldsymbol{v}_r + \boldsymbol{v}_d$$

Where we define

$$\boldsymbol{H}_{sr} \triangleq [(\boldsymbol{H}_{sr,1})^{\boldsymbol{T}}, (\boldsymbol{H}_{sr,2})^{\boldsymbol{T}}, \ldots, (\boldsymbol{H}_{sr,K})^{\boldsymbol{T}}]^{\boldsymbol{T}}$$

$$\boldsymbol{H}_{sr} \triangleq [\boldsymbol{H}_{rd,1}, \boldsymbol{H}_{rd,2}, \ldots, \boldsymbol{H}_{rd,K}]$$

$$\boldsymbol{F} \triangleq \boldsymbol{bd}\,[\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_K]$$

$$\boldsymbol{Vr} \triangleq [(\boldsymbol{v}_{r,1})^{\boldsymbol{T}}, (\boldsymbol{v}_{r,2})^{\boldsymbol{T}}, \ldots, (\boldsymbol{v}_{r,K})^{\boldsymbol{T}}]^{\boldsymbol{T}}$$

Here $(.)^T$ denotes the matrix (vector) transpose, bd(.) stands for a block-diagonal matrix, $\mathbf{H}$sr is a $Nd \; x \; KNr$

channel matrix between all relay nodes and the destination node, Vr is obtained by stacking the noise vectors at all the relays and F is the *KNr x KNr* block diagonal equivalent relay matrix. The diagram of the equivalent MIMO relay system described by (5) is shown in Fig. 2. Thus we see from the above equotation that the received signal vector at the destination can be equivalently written as

$$y_d = \boldsymbol{HW}\hat{s} + \mathbf{v} \quad (6)$$

Where $\boldsymbol{H} = \boldsymbol{H_{rd}FH_{sr}}$is the equivalent MIMO channel and **v**=$\boldsymbol{H_{rd}Fv_r} + \boldsymbol{v_d}$is the equivalent noise.

## 3. Proposed the Source Weight Matrix for MIMO Relay System

We study the following detection algorithms for MIMO relay systems: the ZF algorithm at the transmitter side technique. If we consider the received signal vector at the destination in (4) then our proposed MIMO relay channel (Fig.1) reduces to a MIMO channel (Fig.2) with the equivalent channel matrix of $\boldsymbol{H} = \boldsymbol{H_{rd}FH_{sr}}$ the signal vector of $\hat{s}$ and the equivalent noise vector of **v**=$\boldsymbol{H_{rd}Fv_r} + \boldsymbol{v_d}$.

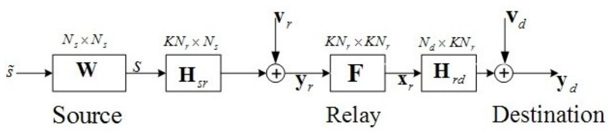


Fig. 2 Equivalent MIMO Channel

Now we can analyze the signal detection Pre-ZF equalization with equivalent MIMO channel.

**The Pre-ZF Algorithm**

The weighting matrix at the transmitter side to be described in this paper is Pre-ZF equalizer. The Pre-ZF equalization can be represented by a Pre-ZF equalizer weight matrix **W** and thus, the precoded symbol vector S can be expressed as

$$s = \boldsymbol{W}\hat{\mathbf{s}} \quad (7)$$

Where ŝ is the original symbol vector for transmission. In case where the ZF equalization is employed, the corresponding weight matrix is given as

$$\boldsymbol{W}_{ZF} = \boldsymbol{Q}_s \times (\boldsymbol{HH^H})^{-1}\boldsymbol{H}^H \quad (8)$$

Where $\boldsymbol{Q}_s$ is a constant to meet the total transmitted power constraint after pre-equalization and it is given as

$$\boldsymbol{Q}_s = \sqrt{\frac{P_s}{tr(\boldsymbol{Q}_s Q_s^H}} \quad (9)$$

Where

$$\boldsymbol{Q}_s = (\boldsymbol{HH^H})^{-1}\boldsymbol{H}^H \quad (10)$$

A simple approach to design the relay is to treat it as an all-pass AF unit, which we construct as F = α$I_{Nr}$, where α is the amplifying factor of the relay and $I_{Nr}$, is an identity matrix of dimension $N_r$ . We can find α from $Pr = \alpha^2 tr\{Ps/Ns(HsrH_{SR}^H + I_{Nr})\}$. Here Ps > 0 and Pr > 0 are the transmit power available at the source and the relay nodes respectively, $(.)^H$ denotes matrix Hermitian and tr{.} indicates trace of matrix.

## 4. Simulation Results and Discussion

In the simulations, the transmission signaling is in spatial multiplexing mode (i.e., the source transmits independent data streams from different antennas) with total transmit power uniformly distributed among the transmit antennas. Also, all simulations are conducted in a flat-fading Reyleigh environment using the BPSK constellation, and the noise variances are assumed to be the same for all antennas. We transmitted $10^3$ randomly generated bits in each channel realization and the BER results are averaged through 200 channel realizations. We plot BER curves versus SNR.

In this example, we simulate Ns = Nr = Nd = 2 and compare the BER performance of the proposed Pre-ZF equalization algorithm with Transmit zero forcing (TxZF) algorithm [15] in MIMO relay channel with varying SNR in the source-to-relay link (SNRs) keeping the relay-to-destination SNR (SNRr) at 20 Db. From Fig. 3, it can be seen that the TxZF algorithm has the worst performance, we achieve a 7 Db gain from TxZF to Pre-ZF in BER = $10^{-2}$.

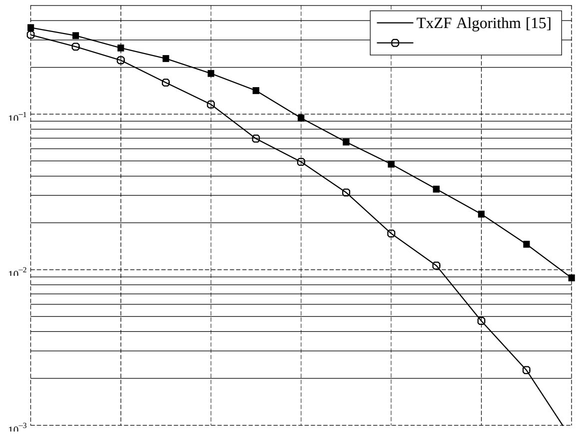


Fig. 3 BER versus SNRs. Ns = Nr = Nd = 2 and SNRr = 20 Db for MIMO relay channel.

## 5. Conculsion

In conclusion, we have demonstrated the advantage of using source weight matrix algorithms for Parallel relay in MIMO relay network. We designed relays as all-pass amplify-and-forward (AF) units which are simpler to implement. Our result demonstrate that Pre-ZF algorithm outperform the TxZF algorithm.

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