BRAGG RESONANCE AROUND THE COAST OF HARDWALL

Viska Noviantri[1]; Ro'fah Nur Rachmawati[2]

[1,2]Mathematics and Statistics Department, School of Computer Science, Bina Nusantara University
Jln. K.H. Syahdan No. 9 Palmerah, Jakarta Barat, 11480
[1]viskanoviantri@binus.ac.id; [2]ashadisalim@binus.ac.id

ABSTRACT

Basically, when waves pass an uneven basis, then this wave will be split into transmission and reflection waves. First of all, it will be shown that a sinusoidal seabed can lead to the phenomenon of Bragg resonance. Bragg resonance occurs when the wave-length comes at twice the wave-length of a sinusoidal basis. The method used to obtain approximation solution is a multi-scale asymptotic expansion method. A research on the effect of Bragg resonance on sinusoidal basis had been studied. Sinusoidal basis can reduce the amplitude of the incoming wave so that the amplitude of the wave transmission is quite small. In these researcher, the coast is assumed ideal and can absorb all the energy of the wave transmission. If the beach can reflect waves, this indicates that the existence of sinusoidal basis is more harmful to the coast. This mechanism relies on the distance between the base sinusoidal and beaches. The present research will examined the influence of the base, when there was a beach of hard-wall on the right, which was perfectly capable of reflecting waves. Having regard to the phase difference, from super positioned waves when they hit the beach, so it can determine the safert and the most dangerous distance.

Keywords: *basic sinusoidal, Bragg resonance, the hard-wall*

INTRODUCTION

The damage of the coast is no longer a strange thing for the public. There are a lot of factors which can cause damages to the coast, either because of human behaviors or because of natural activities. The damage of the coast, caused by natural activities, often happens due to the lunge of sea-water wave that comes with very large amplitudes, for example is the tsunami wave. Basically, when a wave passes an uneven basis, then the wave propagation will be split into two parts, transmission wave and reflection wave. Based on the fact, there are many researchers, who are conducting researches about wave breakers by utilizing the characteristic of the waves. The researches aimed to reduce the amplitude of the waves coming in so the amplitude of transmission wave that heading to the coast is quite small.

The wave breaker can be in the form of numbers of strong and solid blocks with certain sizes, and also the distance between the blocks is adjusted to the needs. The optimum size of the blocks can reduce maximally the amplitude of the wave coming (Heathershaw, 1982).

The uneven seabed can naturally come to existence, and the form of the uneven seabed can split the wave into transmission and reflection waves. Based on this description, the effect of uneven seabed in minimizing the amplitude of the waves, which is transmitted to the coast, will be examined. Bragg resonance gives a significant influence on the size of the amplitude of the wave coming in that is reduced. Bragg resonance happens when a wave propagates through a sinusoidal base that is impermeable and has the number of a wave as much as twice the number of the wave that came (Mei, 2004). The effect of Bragg resonance and the flow upon wave propagation above the porous base has also been examined. Wiryanto (2010) has also examined the unsteady wave propagation through the permeable base.

The phenomenon of Bragg resonance is not able to reduce the amplitude of the wave coming in when the coast, which is located after the sinusoidal base, is a hard-wall. It is going to enlarge the amplitude that coming to the coast, so it can damage the coast. This state can be avoided if the coast has the ability to absorb some waves (Pudjaprasetya & Chendra, 2010). In this research, it will be discussed the basic effect when in the right side, there is a hard-wall coast that can reflect the wave perfectly. Having regard to the phase difference from superposition waves when they hit the beach, so it can determine the distance of the safest and most dangerous. The method, used to obtain the approximation solution, is a multi-scale asymptotic expansion method.

METHODS

The method used in this research is a descriptive method through literature study. However, it is supported by providing the results of the study, which are analytic and numeric analysis. The data used to the simulation is the data from the research that has been previously conducted by Noviantri and Pudjaprasetya (2010). In outline, the method of the research can be seen in Figure 1.

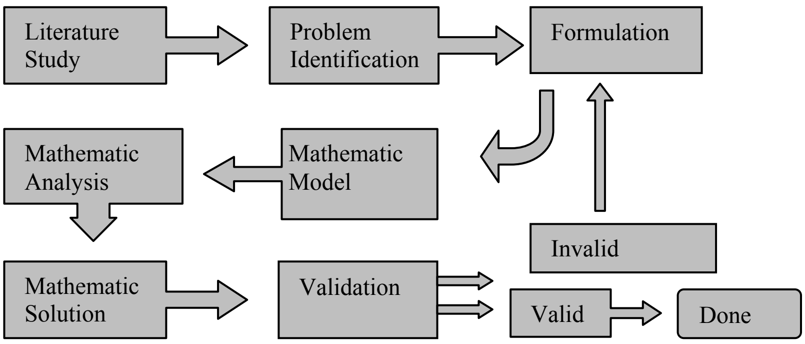


Figure 1 The Research Method Diagram

This research is started with relevant literature studies with the topic of research, either from articles in journals or books, to learn the characteristic of wave propagation. The next stage analyzes the characteristic of wave propagation through sinusoidal basis around the coast of hard-wall. This research is expected to give the image about the distance of the safest and most dangerous between the sinusoidal basis and the coast of hard-wall. Then, an algorithm is made by using Matlab to do a simulation. The data used in the simulation is the data that is obtained from Ref. [6]'s experiment. This simulation was meant to see the tendency change of the safest and most dangerous distance, if the condition of the sinusoidal basis is changed.

RESULTS AND DISCUSSIONS

The amplitude of transmission and reflection wave in the impermeable sinusoidal base are described by supposing that the domain of observations on the sinusoidal basis is $\{z| - h(x) \leq z \leq \eta(x,t)\}$ with h is function one variable that presents the depth of sea water and η is function two variable that presents the deviation wave of the surface of sea water from the state of equilibrium. The function of the depth of water in the surface of sinusoidal sea is

$$h(x) = h_0[1 + \varepsilon D \cos(Kx)] \tag{1}$$

with h_0 states the depth of sea water in the state of equilibrium, or when the surface of sea is flat, $h_0\varepsilon D$ states the amplitude of sinusoidal basis mounds, K states the number of sinusoidal basis wave, and ε is a very small number. Parameter εD is the non-dimension parameter that states the comparison between the amplitude of sinusoidal basis mounds and the depth of sea water in the normal state (h_0).

The equation of one dimension wave in the surface of sinusoidal with a function of depth, as (1) is

$$\frac{1}{g}\frac{\partial^2\eta}{\partial t^2} = h_0\frac{\partial}{\partial x}\left[(1+\varepsilon D\cos Kx)\frac{\partial\eta}{\partial x}\right] \tag{2}$$

By using the method of multi-scale asymptotic expansion, for example

$$\eta(x,\bar{x};t,\bar{t}) = \eta_0(x,\bar{x};t,\bar{t}) + \varepsilon\eta_1(x,\bar{x};t,\bar{t}) + \varepsilon^2\eta_2(x,\bar{x};t,\bar{t}) + \cdots \tag{3}$$

Substitute (3) to (2), so it is obtained a solution of the quarters in the order 1 from the equation 2, is

$$\eta_0 = \frac{A(\bar{x},\bar{t})}{2}e^{ikx-i\omega t} + c.c + \frac{B(\bar{x},\bar{t})}{2}e^{-ikx-i\omega t} + c.c, \tag{4}$$

with c.c is the complex conjugate, A and B are the function 2 variable that depend on x and t.

Notice that $A\,(x;t)/2$ and $B\,(x;t)/2$ are consecutively the envelopes for the waves that propagate to the right and to the left. The size of the amplitude of waves, that propagate to the right and to the left directly, can be obtained through the sizes of $A\,(x;\,t)$ and $B\,(x;\,t)$. Meanwhile, the quarters that have $O(\varepsilon)$ in the equation (2) are

$$2h_0\frac{\partial^2\eta_0}{\partial\bar{x}\partial x} + h_0\frac{\partial^2\eta_1}{\partial x^2} + h_0 D\frac{\partial}{\partial x}\left(\cos Kx\frac{\partial\eta_0}{\partial x}\right) = \frac{2}{g}\frac{\partial^2\eta_0}{\partial\bar{t}\partial t} + \frac{1}{g}\frac{\partial^2\eta_1}{\partial t^2},$$

$$h_0\frac{\partial^2\eta_1}{\partial x^2} - \frac{1}{g}\frac{\partial^2\eta_1}{\partial t^2} = \frac{2}{g}\frac{\partial^2\eta_0}{\partial\bar{t}\partial t} - 2h_0\frac{\partial^2\eta_0}{\partial\bar{x}\partial x} - h_0 D\frac{\partial}{\partial x}\left[\frac{1}{2}\left(e^{iKx}+e^{-iKx}\right)\frac{\partial\eta_0}{\partial x}\right]\ldots \tag{5}$$

Substitute $K = 2k$ and the equation (4) and (5), then simplified, so it is obtained

$$h_0\frac{\partial^2\eta_1}{\partial x^2} - \frac{1}{g}\frac{\partial^2\eta_1}{\partial t^2} = \frac{1}{g}\left[\frac{\partial A}{\partial\bar{t}}(-i\omega)e^{-ikx-i\omega t} + c.c + \frac{\partial B}{\partial t}(-i\omega)e^{-ikx-i\omega t} + c,\right]$$

$$-ho\left[\frac{\partial A}{\partial\bar{t}}(-i\omega)e^{ikx-i\omega t} + c.c + \frac{\partial B}{\partial\bar{x}}(-ik)e^{-ikx-i\omega t} + c.c\right]$$

$$-\frac{h_0 D}{4}\frac{\partial}{\partial x}\left[\left(e^{2ikx-i\omega t} + e^{-2ikx}\right)\frac{\partial}{\partial x}\left(Ae^{ikx-i\omega t} + c.c + Be^{-ikx-i\omega t} + c.c\right)\right] \tag{6}$$

The last quarter on the right side of the equation (6) can be simplified to

$$-\frac{h_0 D}{4}\left(k^2 B e^{ikx-i\omega t} + c.c + k^2 A e^{-ikx-i\omega t} + c.c - 3k^2 A e^{3ikx-i\omega t} + c.c - 3k^2 B e^{-3ikx-i\omega t} + c.c\right)$$

To avoid an unlimited resonance and to guarantee a solution for $\eta 1$, then the coefficients $exp\ (ikx - \omega t)\ and\ exp\ (ikx + \omega t)$ on the right side (26) should be made zero and through the algebra manipulation, it can be obtained

$$\frac{\partial^2 A(\bar{x},\bar{t})}{\partial \bar{t}^2} - c^2 \frac{\partial^2 A(\bar{x},\bar{t})}{\partial \bar{x}^2} + (\Omega_0)^2 A(\bar{x},\bar{t}) = 0, \quad (7)$$

To be clearer, the equations that apply to each areas are provided by Table 1.

Table 1 The Scheme of Equations *A* and *B,* Also the Validity of the Area.

Region	Equation
$\bar{x} < 0$	$\frac{\partial A}{\partial \bar{t}} + c\frac{\partial A}{\partial \bar{x}} = 0, \frac{\partial B}{\partial \bar{t}} - c\frac{\partial B}{\partial \bar{x}} = 0$
	$c\frac{\partial A(\bar{x},\bar{t})}{\partial \bar{x}} + \frac{\partial A(\bar{x},\bar{t})}{\partial \bar{t}} = \frac{ikcD}{4}B(\bar{x},\bar{t}),$
$0 < \bar{x} < \bar{L}$	$\frac{\partial B(\bar{x},\bar{t})}{\partial \bar{t}} - c\frac{\partial B(\bar{x},\bar{t})}{\partial \bar{x}} = \frac{ikcD}{4}B(\bar{x},\bar{t}),$
$\bar{x} > \bar{L}$	$\frac{\partial A}{\partial \bar{t}} + c\frac{\partial A}{\partial \bar{x}} = 0, B = 0.$

Characteristic of the coast is in respect to the fact that every beach has abilities to absorb or to reflect the wave that coming in. The coast that only consists of sands has the ability to absorb quite big waves (small reflected power), meanwhile the coast that consists of hard walls can perfectly reflect the waves. Besides the material of coast constituent, the form of the coast also affects the size of wave reflection. In fact, there is no coast that can perfectly reflect of absorb the waves. For more detail, pay attention to Figure 2 and 3.

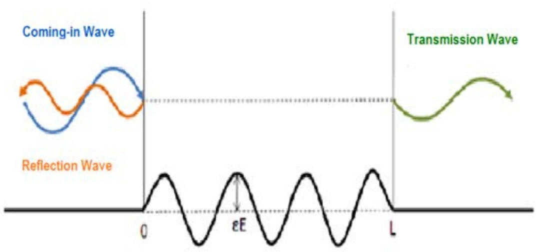


Figure 2 The Coast With the Ability to Perfectly Absorb Waves

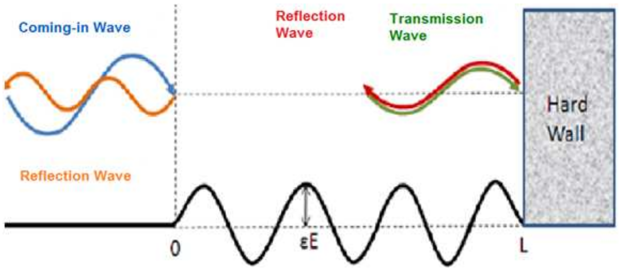


Figure 3 The Coast With the Ability to Perfectly Reflect Waves

For example, in the position $x = L$ (at the end of sinusoidal basis), there is a coast. If A and B consecutively stated the amplitudes that are moving to the right and to the left, $|R|$ stated the proportion of the amplitude of reflected waves, and θ is the phase difference between the coming-in wave and the reflected wave, then the abilities of the coasts to absorb or to reflect the waves are represented by the following equation

$$\frac{B(L,t)}{A(L,t)} = |R|e^{i\theta}. \quad (8)$$

Thus, the characteristics of the coast are affecting to the right condition in $x = L$. The right condition to the three types of coast can be seen in the Table 2.

Table 2 The Right Condition in $x = L$ for Several Characteristics of Coast

Type of Coast	Phase Difference Between A and B ($\boldsymbol{\theta}$)	Right Limit Condition
Perfect absorb		$B(L,t) = 0$
Perfect reflect	0	$B(L,t) = A(L,t)$
	$\boldsymbol{\pi}$	

Numerical discretization concerns about the influence of the coast against Bragg resonance can be seen through a numerical approach. Therefore, it is going to be explained the method of numerical approximation, that is used to discrete the equation of waves previously obtained. Previously, it has been described that the change of the amplitude of monochromatic waves that is going in from the left, and experiences Bragg resonance due to the sinusoidal basis, determined by the system of differential equations as follows:

$$c\frac{\partial A(\overline{x},\overline{t})}{\partial \overline{x}} + \frac{\partial A(\overline{x},\overline{t})}{\partial \overline{t}} = \frac{ikcD}{4}B(\overline{x},\overline{t}),$$
$$\frac{\partial B(\overline{x},\overline{t})}{\partial \overline{t}} - c\frac{\partial B(\overline{x},\overline{t})}{\partial \overline{x}} = \frac{ikcD}{4}A(\overline{x},\overline{t}). \quad (9)$$

where the initial condition $A\ (x,0) = 0$ and $B\ (x,0) = 0$, also the limit condition $A\ (0;\ t) = A_0$. Meanwhile the limit condition B (L, t) depends on the characteristic of the coast.

To see the behavior of the waves amplitude above the sinusoidal basis that propagates to the right (A) and to the left (B), the system of equation (9) will be discreteed with the method of different-up-to. The first equation from (9) is the equation transport for the waves that propagate to the right, therefore this equation is distinguished by using the FTBS scheme. Meanwhile, the second equation is unattached to the FTFS scheme.

Thus, it is obtained two different equations, as follow:

$$\mathrm{A}_j^{n+1} = (1-r)\mathrm{A}_j^{n} + r\,\mathrm{A}_{j-1}^{n} + \beta \mathrm{B}_j^{n}$$

$$\mathrm{B}_j^{n+1} = (1-r)\mathrm{B}_j^{n} + r\,\mathrm{B}_{j-1}^{n} + \beta \mathrm{A}_j^{n} \tag{10}$$

with r and β, as follow

$$r = c_1\frac{\Delta t}{\Delta x}, \frac{kcD}{4} = \frac{\omega D}{4}$$

Later on, there is a simulation of the influence of coast to Bragg resonance to the condition of hard-wall coast.

To make the different-up-to scheme stable, then an analysis of stability should be conducted to determine the value of r. The different-up-to scheme is stable if the value of amplification factor, ρ is less than, or equal to 1. First of all, from the equation (10), it can be obtained an equation that depends only on A or only B, which is (11)

$$2(1-r)\mathrm{B}_j^{n+1} - (2r^2 - 2r + 1 + \alpha^2)\mathrm{B}_j^{n} - \mathrm{B}_j^{n+2} + r\left(\mathrm{B}_{j-1}^{n+1} + \mathrm{B}_{j+1}^{n+1}\right)$$
$$-(1-r)r\left(\mathrm{B}_{j-1}^{n} + \mathrm{B}_{j+1}^{n}\right) = 0. \tag{11}$$

A similar equation is also obtained for A. To obtain the condition of stability from the different-up-to scheme, an analysis of Van Neumann is going to be used, by firstly giving example of (12)

$$\mathrm{B}_j^{n} = \rho^n e^{iaj}. \tag{12}$$

Then, do the substitution (12) into the different-up-to scheme (10), so at the end it will be obtained that the scheme will be stable to the following Courant number

$$r = c1\frac{\Delta t}{\Delta x} \leq 1. \tag{13}$$

Even though the FTBS scheme and FTFS scheme are the order-1 schemes, however these schemes can produce a quite good approximation solution by choosing Δx and Δt that fulfill the Courant numbers,

$$r = c1\frac{\Delta t}{\Delta x} = 1. \tag{14}$$

The amplitude of the waves that hits hardwall can be imagined as a monochromatic wave with the amplitude A_0 and the number of the wave k_1, that is consistent with the length of the wave $\lambda_1 = 2$m entering the sinusoidal basis with the length of L = 10m, and the number of the wave K, that is consistent with the length of the wave λ = 1m. According to the simulation data that had been given, the following is going to show the result of numerical simulation to the type of hard-wall coast, which perfectly reflects the waves with phase difference 0. From the result of this simulation, it will be known how large the amplitude of transmission wave that hits the hard-wall coast is.

Around the coast that caused B (L,t) = A (L,t), the behavior of transmission and reflection waves above the sinusoidal basis can be seen on Figure 4. In Figure 4, the green, blue and red lines consecutively states the amplitude of the wave of transmission A, energy flux, and the amplitude of the wave of reflection B that is provided to several observation times. For $t \leq 2$, a similar explanation with both previous cases also applies in this case. When $t = 2.5$, the effect of new coast is felt in some areas of the sinusoidal basis, which is at $x > 7.5$. In this area, the amplitude of reflection wave is equal to the amplitude of transmission wave because the coast reflects the waves perfectly without turning phase. When $t = 6$, the effect of the coast has been felt along the sinusoidal basis, so that almost in every x. The amplitude of the reflection waves worth almost the same as the amplitude of the transmission waves.

There has been no more transfer of energy between A and B after $t = 10$, so the values of A and B has no longer been changing towards the position. The transfer of energy takes longer time than both of the previous cases. At the end, it is obtained that the amplitude of the coming-in waves has not been reduced at all. Even the transmission waves, that towards the coast, have more than twice amplitudes than the amplitude of the coming-in waves for $''\varepsilon D = 0.8$. It is clearly dangerous for the coast. In this case, it can be stated that the transmission waves and the reflection waves around the coast are mutually reinforcing, that often called the constructive superposition. This constructive superposition causes the amplitude of the transmission waves is way bigger than the amplitude of the coming-in waves. Of course, this large amplitude can pose a threat to humans who live near the coast. According to this result, it can be known that the sinusoidal basis around the coast, which has certain characteristic, is not always beneficial for the coast.

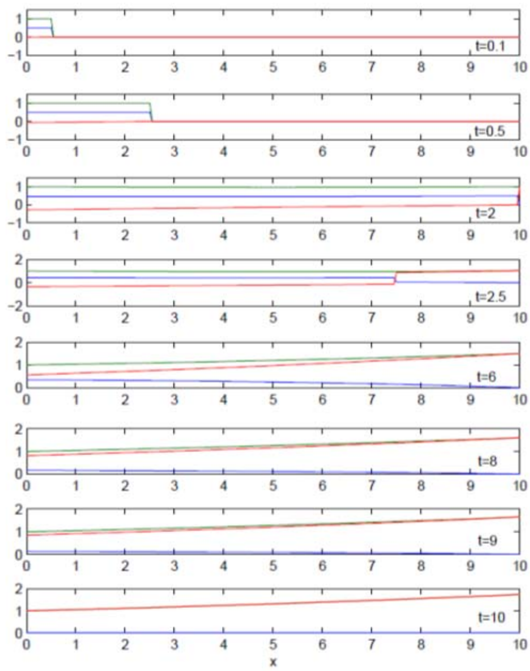


Figure 4 Graphic $A(x;t)$: Green, $B(x;t)$: Red, and Energy Flux: Blue, for $B(L;t) = -A(L,t), \varepsilon E = 0.08B$

The Optimal Length of Sinusoidal Basis is explained in Figure 5 as a material in a form of sinusoidal is submerged with the amplitude εDh_0, and is located in the fluid basis with the depth of h_0. The function of the depth of fluid can be stated as

$$h(x)=\begin{cases} h_0-\varepsilon Dh_0, & 0\le x\le L/2\\ h_0+\varepsilon Dh_0, & \mathrm{L/2}\le \mathrm{x}\le \mathrm{L}\end{cases} \tag{15}$$

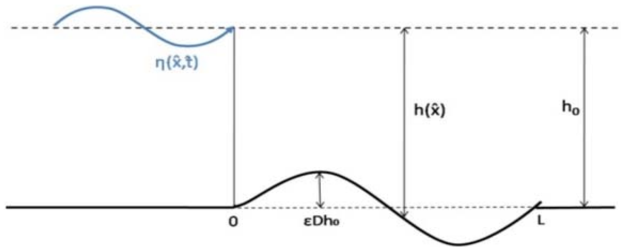


Figure 5 The Mound of Sinusoidal 1 Basis

Based on previous research [7], the optimal length from the sinusoidal basis (0 ≤ x ≤ L/2) is the following

$$L_{optimal}=\begin{cases} \dfrac{\pi}{2\omega}\sqrt{gh_0(1-\varepsilon D)}, & 0\le x\le L/2\\ \dfrac{\pi}{2\omega}\sqrt{gh_0(1+\varepsilon D)}, & \mathrm{L/2}\le \mathrm{x}\le \mathrm{L}\end{cases} \tag{16}$$

Therefore, the length of the sinusoidal basis waves optimally is

$$L_{optimal}=\frac{\pi}{2\omega}\sqrt{gh_0(1-\varepsilon D)}+\frac{\pi}{2\omega}\sqrt{gh_0(1+\varepsilon D)} \tag{17}$$

The constructive and destructive Superpositions can be described correspondingly in accordance to the previous description, it is known that the existence of the sinusoidal basis around the hard-wall coast (the coast that is usually composed from hard stones and very steep) can be dangerous. The amplitude of the transmission waves that is coming to the coast can be twice bigger than the original amplitude. In the next description, the hard-wall coast with the distance L_p from the row n of the sinusoidal basis will be explained.

Knowing the previous figure, make an assumption that the sinusoidal basis has the optimal length $L_{optimal}$ (L_0). The first transmission wave has passed $x = L_0$ with a notation $A_1\exp[i(k_0x - \omega t))$ will move to the right, and when it hits the coast, the waves will phase return at π becomes the reflection waves. This reflection wave, when reaching $x = L_0$ will split into two, the wave that stays moving to the left, and the wave that moves to the right. The wave that moves to the right next will become the second transmission wave that towards the coast. This wave is denoted as $A_2\exp[i(k_0(x + 2Lp)] + \pi - \omega t)$ when it reaches the coast. This reflection and transmission process is consecutively going to repeat, so the n wave that reaches $x = L_0 + L_p$ has a synthesis in the form

$$A_n e^{i(k_0(x+2(n-1)Lp)+\pi-\omega t)} \quad \text{with n = 1,2,3,....} \tag{18}$$

As the time goes by, the waves that arrives at $x = L_0 + Lp$ is the superposition that mutually weaken if the phase difference between the wave $k02L_0+ \pi$, is the multiple of odd numbers from π.

Therefore, the safe distance of the coast is

$$(k_0(x+2L_p)+\pi-\omega t)-(k_0x-\omega t)=(2n+1)\pi$$

$$\begin{aligned} k_0 2L_p+\pi &= (2n+1)\pi \\ \frac{2\pi}{\lambda_0}2L_p &= 2n\pi \\ 2L_p &= n\lambda_0 \end{aligned} \tag{19}$$

with n = 1,2,3,….

On the contrary, if the phase difference between the wave, which is $k_0 2L+\pi$, is the multiplication of even numbers from π or

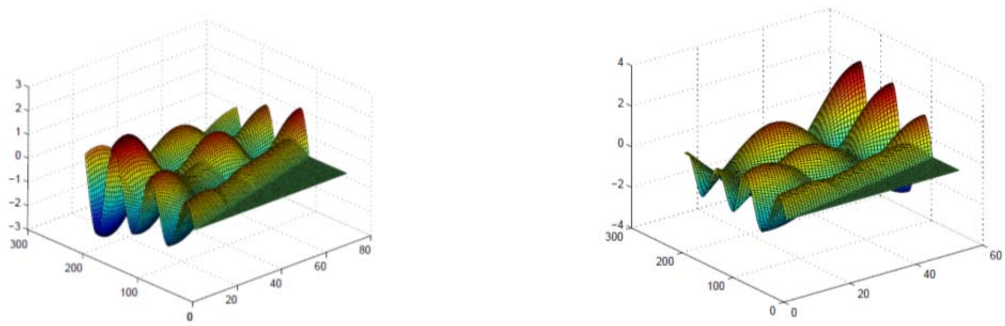
$$2L_p = \left(n\text{-}\frac{1}{2}\right)\lambda_0 \tag{20}$$

with n = 1,2,3,…. Then the waves at $x = Lo + Lp$ will mutually strengthen superposition. This condition enlarges the amplitude of the waves that hit the coast.

In this opportunity, wave breaker 1-mound of sinusoidal it is going to be simulated through the events of the entrance of monochromatic wave from the left that later moves toward the hard wall coast in the right. This wave, through the wave breaker, is 1-mound of sinusoidal. The parameters that will be used is as follow. The gravity g = 10, the frequent of wave $\omega = 3\pi/10$, the amplitude of monochromatic wave that is coming in A = 1, and the basis is a function

$$h(x) = 1 + 0.08\cos(Kx) \tag{21}$$

then the optimal length of sinusoidal basis is $L_{optimal}$ = 10.



(a) (b)

Figure 6 The Surface of Wave Passes the Sinusoidal Basis 1-Mound
In (a) The safest distance and (b) The most dangerous distance

Pay attention to Figure 6(a), it can be observed that as the time passes, the superpositions that mutually weaken happened, so the amplitude of the wave, that arrived at the coast, is getting smaller. At the first time the wave hits the coast, the amplitude is about 2, with increasing time, the amplitude is about 1*;* 3. In contrast, in Figure 6(b), the superposition that mutually strengthen happened, so the amplitude of the original wave is about 2, and getting bigger until it reaches more than dax 3. This means that if the sinusoidal basis is located in the dangerous distance from the coast, then its existence is enlarging the amplitude of the waves that are transmitted to the coast.

CONCLUSIONS

The existence of sinusoidal basis can reduce the amplitude of the coming-in waves if it is around the coast that can absorb the waves perfectly or partially. Likewise the thing if the coast can reflect the wave with the phase difference π, then a superposition that mutually weaken will happen, so it will not dangerous for the coast. However, it is not the same if the said coast, was the hard-wall coast that is usually composed of very hard stones. The ability of hard-wall that can reflect the waves perfectly with the same phase, precisely gives huge negative impacts because the occurence of superpositions that mutually strengthen. This can be avoided by controlling the distance between the sinusoidal basis and the coast. This research has given a result of the model for the optimal distance or the safest distance between the sinusoidal bases, that consists of one mound with the coast, so the amplitude of the transmission wave that comes toward the coast is minimal. The most dangerous distance is also successfully modeled, so the bad impacts of the hard-wall can be avoided.

REFERENCES

Heathershaw, A. D. (1982). Seabed-wave resonance and sandbar growth. *Nature*, *296*, 343-345. doi: 10.1038/296343a0

Mei, C. C. (2004). *Multiple Scattering by an Extended Region of Inhomogeneities.* Retrieved from http://web.mit.edu/1.138j/www/material/chap-5.pdf.

Noviantri, V. & Pudjaprasetya, S. R. (2010). The Relevance of Wavy Beds as Shoreline Protection. *Proceedings of the 13th Asian Congress of Fluid Mechanics,* 489-492.

Pudjaprasetya, S. R. & Chendra, H. D. (2010). An Optimal Dimension of Submerged Parallel Bars as a Wave Reflector. *The Bulletin of the Malaysian Mathematical Society*, *32*(1), 55-62.

Wiryanto, L. H. (2010). Unsteady Waves Generated by Flow over a Porous Layer. *IAENG International Journal of Applied Mathematics*, *40(*4).