THE CARTESIAN CUT-CELL METHOD IN TWO DIMENSIONAL FLOOD SIMULATION

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Abstract

Two dimensional dynamic models have been increasingly used for river flood simulation. This commonly uses satellite remote sensor data, recorded on a rectangular (Raster) grid. There are many important features on a flood plain, such as hedges or buildings, which do not follow the grid lines. Irregular meshes can be used to follow these features, but converting raster data to this format involves a loss of detail. The Cartesian cut-cell (CC) method uses a rectangular mesh. The edges of irregular solid bodies are located precisely with sequences of vertex coordinates. Cut-cells, which lie on the edge, are given special treatment. This allows straightforward integration of grid and vector data, potentially within a GIS based framework. This paper introduces the semi permeable internal (SPIn) boundary cut-cell method. This allows the integration of permeable boundaries, such as hedges, into the model. To explore the impact of these features, a small scale river flood event, over a field featuring a hedgerow, is simulated.

Keyword: raster grid, irregular meshed, GIS, semi-permeable boundary

1. Introduction

In mapping flood risk 2-dimensional hydraulic models are coming to be value increasingly as a tool to account for spatial variations in out of channel flows. Advances in computer power have made these techniques feasible, but it is the ever increasing store of high quality geographical data which makes them so broadly applicable. Satellite remote sensors provide detailed data on the surface elevation, land use and water coverage (Cobby et al [4], Townsend & Walsh [9]). The method of data collection yields a regular orthogonal raster grid of point values. The level of detail provided is particularly valuable for floodplains where small changes in elevation can easily be missed by standard techniques.

Information on land use and other features is also valuable, but using this data effectively can be problematic. Extensive empirical studies have been conducted into the relation between vegetation and effective bed roughness (Werner et al [10]). However, the effective roughness can vary considerably within a site, seasonally, and even during a single flood event. The heterogeneity of land use has been of particular interest to those modelling floodplain flows in two dimensions (Cobby et al [5], Romanovics et al [8}). Calibration of roughness parameter has historically relied on relatively few measurements of flood events, such as river level gauges which can be unreliable at high flows (Pappenberger et al [7]), and post event observations of flood damage of debris. This has led to simplifying the classification of land use (Aronica et al [1], Grayson et al [6]). Satellite images showing water surfaces during flood events, have had limited value in enabling a more spatially diverse model of land features (Bates et al [2], Werner et al [10]). Where sensitivity to distributed sets of roughness parameters has been shown, floodplain flow has formed a significant component in overall conveyance. It is expected that flow routing has a major influence on such events, and that linear land features, as well as topography, can have a profound impact on this.

Two dimensional hydraulic models often use and irregular mesh. These have been adapted to better represent particular manmade features, such as hedgerows or roads, and to pick out important aspects of topography (Cobby et al [5}). In representing raster data, individual grid points may be selected. This involves discarding valuable data on the discarded grid point, and may still fail to accurately place features which do not lie neatly on the grid. Alternatively, levels may be interpolated at any point, which necessarily involves loss of definition through smoothing.

Orthogonal meshes are simpler to implement and allow the use of certain numerical

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techniques not possible otherwise. It also allowed the mesh to be fitted to the raster grid, allowing direct use of this data. However, accuracy in the placement of features is almost always compromised.

Cartesian cut-cell method (Causon et all [3]) define irregular solid bodies on a rectangular mesh. Special treatment is given to the cut-cell on the edge of solid body. This paper present extension of the cut-cells method to encompasses permeable (Pin), and semi-permeable (Spin) internal boundaries. This would allow accurate placement of a range of features regardless of where they lie relative to the grid. It is also a methodology which can be readily integrated with established GIS protocols.

A test case demonstrates the use of cut-cells and polygon defined regions, illustrating the impact of hedgerows on floodplain flows. AMAZON-CC, a numerical scheme based on an explicit Godunov type finite-volume approximate Riemann solver, (Zhou et al [13] is used, which is 2nd order accurate in time and space.

2. The Cartesian Cut-Cell Method Development and Context

The Cartesian cut-cell (CC) method was developed for use with finite volume methods. It accurately describes the boundary of a solid body within a regular Cartesian grid as an irregular polygon with a sequence of vertices within domain (Figure-1). Those cut cells which are closed by the boundary are classified according to their shape and the orientation of solid wall. To preserve the time step without compromising stability, smaller cells are merged with larger neighbouring cells.
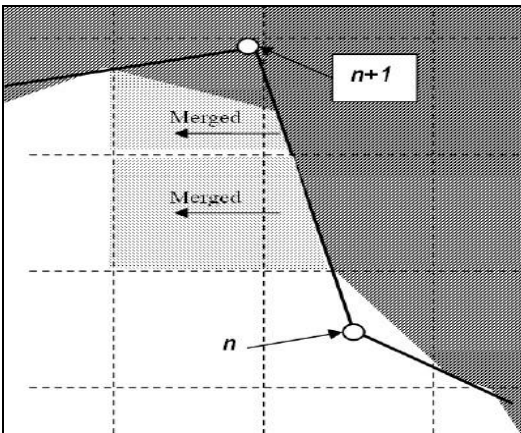

Figure 1. Section of Cartesian mesh with solid areas

A section of Cartesian mesh with solid areas derived from the intersection points between polygon edges and grid lines. Solid cells and solid areas of cut-cells are shaded. Vertices are listed anti-clockwise around the body. Smaller cells are merged with their neighbour across the longest edge. Finite volume methods find the flux across the interface between two cells based on an estimation of the variables on either side. Excepting where discontinuities occur, for normal cells in CC mesh it is possible to accurately reconstruct local gradients, based on neighbouring cell values, to achieve a second order accurate estimate of these fluxes.

The surface Gradient Method (Zhou et al [11]) is applied to ensure the bed slope balances the flux gradient. A slope limiting function ensures that where a discontinuity does occur, this does not lead to unwanted non-physical solutions. While a second order accuracy in space is harder to achieve for cut cells, the proportion of cells which are cut decreases as the grid resolution becomes finer. Thus, while there is some compromise in accuracy, the total contribution of errors from these cells is consistent with that from the reminder of the domain.

It is a natural extension to these techniques, to have regions defined in the same way as solid bodies, but with a permeable boundary. The method for calculating the flux across a solid boundary uses ghost point values, so using actual values on either side of a permeable internal (PIn) boundary is feasible. Also, the numerical methods used here have been applied successfully on irregular meshes, although without the same spatial accuracy. Since the CC mesh can be considered a special case of the irregular mesh, similar success can be expected.

3. The Effects of Semi-Permeable Internal Boundaries on Floodplain Flow

It is too early in the development of the methods described above, for validation. Instead, a simple case of overbank flows on a small scale is used to show that the finite volume methods used can give realistic results. It is also explores the impact that the flow resistance and placement of hedges can have on floodplain flows.

Flow travels from bottom left to bottom right. Solid lines are impermeable. Dashed lines indicate a change in bed level or gradient. The hedge is shown as a mottled region its default position. The domain (figure-2) consist of a rectangular channel 10 m

long by 1 m wide, with a 1:100 slope descending from West to East in the direction of flow. The south side has a solid wall, with a solid trapezoid obstacle projecting 0.3 m into the stream halfway along. The north side has a slope leading up to a floodplain, which is separated into Eastern and western regions by a hedge. The Chezy roughness parameters for the channel is 100, west of the hedge is 40, east of the hedge is 20, and the hedge has value of 2. Strict inflow and outflow conditions (table-1) are enforced to ensure the same overall mass balance.

Table 1. Default discharge at entry to and exit from channel. Discharge changes linearly between stages.

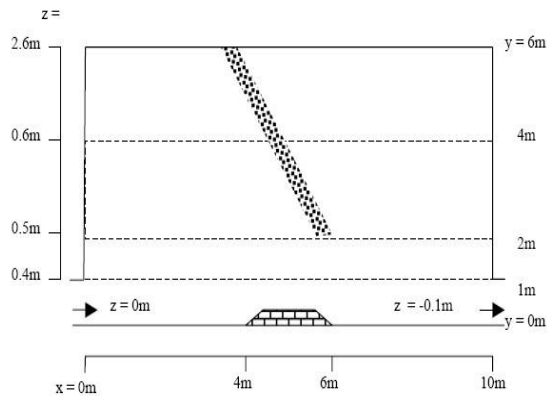| Time (inflow) (seconds)	Time (outflow) (seconds)	Discharge (Cumecs)
0 – 120	0 – 480	0.03
120 – 420	480 – 780	Increasing
420 – 900	780 – 1260	0.04
900 – 1200	1260 – 1560	Decreasing
After 1200	After 1560	0.03

Figure 2. A map of the test case.

Sensitivity analysis was carried out for roughness parameters and the location of the hedge. The roughness parameters were changed by 20 %. The ends of the hedge were moved 2 metres either east or west. This should be taken into account when evaluating the impact. The volume of water out of the channel is used for comparison. Root mean square (RMS) difference between time series and the difference between maximum volumes of the control and test cases are given in table-2. The

maximum volume of inundation was 1.186 cubic meters after approximately 11 minutes from the start of the simulation. This varied by less than 0.4 % in all the tests. The RMS difference was more significant, the intersection between floodplain and channel being greater during inundation than recession. Figure-3 and Figure-4 show the peak flood flows.

The hedge appears to influence flows in two distinct ways. Locally it retards the flow, but it can also help the channel these flows. This flow routing is particularly strong when the southern end of the hedge is close to the channel obstacle, as these both serve to constrict the flow. The degree to which this flow is diverted away from the channel does not only depend on the location of the hedge.
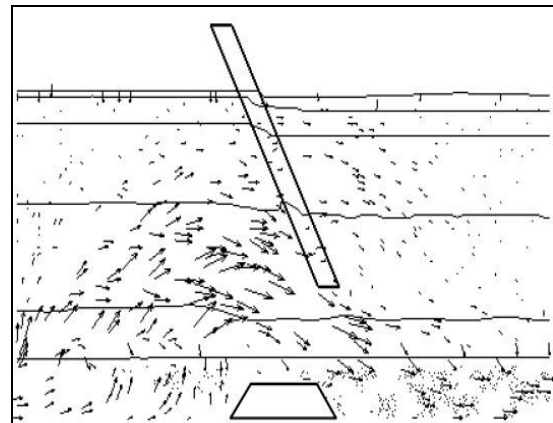

Figure 3. Default case at maximum inundation.

The figure above shows the flow field with arrow length proportional to velocity. The hedge and channel obstacle are shown in outline. Contours indicate depth of 0 m, 0.02 m, 0.05 m, 0.1 m, and 0.2 m. The result could be different if the hedge moved 2 meters east.
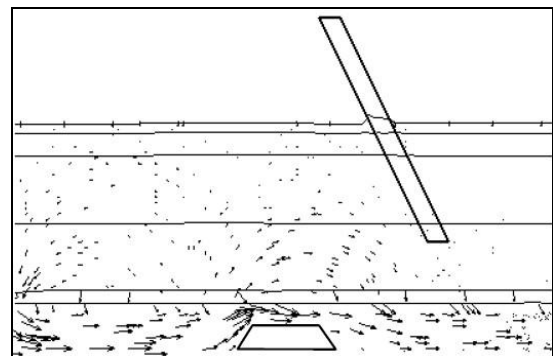

Figure 4. Test case with hedge moved 2 meters east at maximum inundation.

Table 2. The result of test case with different hedge

Region	Chezy value for roughness	Difference in Floodplain Storage (cc)	
		RMS	At Peak
Channel	120	0.37	-0.32
	80	0.41	-0.48
West end	50	1.11	-0.68
	30	1.71	-0.08
East end	25	1.17	0.24
	15	1.50	-0.08
Hedge	2.5	3.83	1.92
	1.5	2.84	2.56
South of hedge (m)	North of hedge (m)		
7.6 – 8.0	3.6 – 4.0	14.54	0.12
3.6 – 4.0	3.6 – 4.0	2.81	-4.20
5.6 – 6.0	5.6 – 6.0	3.80	3.63
5.6 – 6.0	1.6 – 2.0	1.64	1.00
7.6 – 8.0	5.6 - 6.0	3.29	-2.32
3.6 – 4.0	1.6 – 2.0	2.62	-3.92

The first two columns show the parameter altered and its new value. The last two columns give measures of difference between the test results and the results from the default case. Both increasing and decreasing the Chezy value of the hedge increased maximum inundation. This may indicate that this value influences both the retarding and the routing effects. The degree to which flow was constrained, by the solid barrier and by the constricted discharge at the eastern end of the domain, probably also contributes to these phenomena.

4. Conclusions

The methods used give a reasonably convincing simulation of flood flows, though without comparison with a real event, no definite evaluation can be made. The strong influence of the hedgerows on the direction of flow indicates that the accurate placement of such features may well be significant. Since a major motivation behind the use of two-dimensional simulations is to deal with the spatially diverse flow fields that have been observed on floodplains, accuracy in placing features which influence these must be important. PIn ans SPIn boundaries are potentially useful tools for achieving this.

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