Effect of Energy Gain and Loss in Breathing Pattern of Solitary Wave for Nonlinear Equation

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Abstract— Nonlinear phenomena like soliton propagate over long distance in transmit information, without dispersion energy due to the properties of the solitons, which has balanced of the nonlinearity effect and dispersion effect resulted the signal undistorted and symmetric bell shape curve. We study about the properties and breathing pattern of solitary wave of pulses in absence and present of energy loss, by using one dimensional nonlinear equation; cubic-quintic complex Ginzburg-Landau equation (cqCGLE). Breathing pattern of soliton behaviour is constructed with hyperbolic sine and hyperbolic tangent as initial amplitude profile and observed by means of numerical simulation. Resulting in observation of breathing pattern of soliton in term of energy loss and gain while travelling, but it still maintains spatial localization of wave energy in the changing pulses shape through a unique dissipative soliton.

Keywords— Soliton, nonlinearity, dispersion, breathing, energy.

1 INTRODUCTION

In classical soliton theory, soliton was originally used to refer to localized solutions of integrable nonlinear system. These integrable solitons do not change their shape and velocity after colliding with each other, and they remain intact when interacting with radiation waves. Integrable conventional solitons result from the single balance between nonlinearity and dispersion or diffraction in Hamiltonian system.

But then, in early 1990s, physicists realized that solitary waves did exist in a wide range of non-integrable and non-conservative systems, this time, a dramatic turning point occurred. New terminology needed to be introduced, and thus solitary waves in nonlinear optical systems with nonlinear gain or loss mechanisms were thereafter referred to as dissipative solitons. The waves result due to perturbations that cause the soliton to lose some energy.

The purpose of the present work is to observe the effect of energy gain and loss in breathing pattern of solitary waves. In order to produce stable localized stationary solutions that arise in nonlinear spatially extended dissipative systems, gain and loss of energy must be balance, with small effect on perturbation, where energy, profile and chirp of a dissipative soliton are predetermined by the equation parameters, rather than by the perturbation on its initial conditions.

Based on ideas of Prigogine in open systems which also decreases its thermodynamical entropy must necessarily export or dissipate such entropy to its surroundings in order to maintain their organization as dissipative structures and far from equilibrium; ‘dissipative systems’ occur when there is exchange of energy with their environment, and this can manifest conditions for self-organization [1]. This self-organized formations requiring a continuous supply of matter or energy. As soon as that supply finishes, a dissipative soliton ceases to exist.

A stationary dissipative soliton therefore results from the continuous exchange of energy with the environment along with the live redistribution of energy between various parts of the soliton. The area of consumption and dissipation of energy are both frequency and intensity dependent.

Although the profile of a dissipative soliton is indeed fixed for a given set of equation parameters, we can obtain a wide range of profiles and various transformations of dissipative solitons in the form of bifurcation by changing these parameters.

We present the numerical simulation by introducing the nonlinear wave equation along with initial condition and numerically observe behaviour of soliton breathing patterns in term of interaction with environment in the present of loss and gain of energy.

2 METHOD

2.1 Equation and Parameters

Although the profile of a dissipative soliton is indeed fixed for a given set of equation parameters, a wide range of profiles can be obtained by tuning these parameters. Because the optical field is a function of continuous spatial variables, the related dynamical system possesses infinite degrees of freedom. We can observe the variation evolution along their direction of propagation in shape, amplitude and width of the wave.

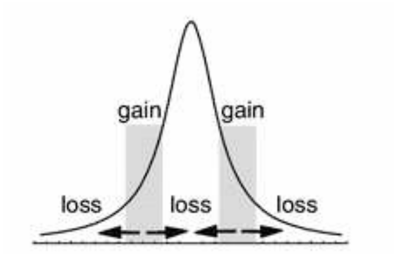


Fig. 1. Qualitative description of solitons in dissipative system.
The soliton has area of consumption as well as expenditure of energy and these can both frequency (spatial or temporal) and intensity dependent. Arrows show the energy flow across the solitons.

The one-dimensional (1D) nonlinear cubic quintic complex Ginzburg-Landau equation has been considerable attention in diverse branches of science in dissipative system. The formation can move with constant velocity when the matter supply is too scarce for stationary but when stationary state does not provides a sufficient rate of energy supply, our dissipative solitons can oscillate.

We start our analysis with the cubic-quintic complex Ginzburg-Landau equation, an ideal-typical model for dissipative solitons in the form of

$$i\frac{\partial\psi(t,x)}{\partial t}+\left(\frac{D}{2}-i\beta\right)\Delta\psi+\gamma|\psi|^2\psi+\nu|\psi|^4\psi=i\delta\psi+i\varepsilon|\psi|^2\psi+i\mu|\psi|^4\psi \quad (1)$$

For one-dimensional cubic-quintic complex Ginzburg-Landau equation [1]-[3], presented as

$$i\frac{\partial\psi(t,x)}{\partial t}+\left(\frac{D}{2}-i\beta\right)\psi_{xx}+\gamma|\psi|^2\psi+\nu|\psi|^4\psi=i\delta\psi+i\varepsilon|\psi|^2\psi+i\mu|\psi|^4\psi \quad (2)$$

TABLE 1. UNITS FOR NONLINIER CUBIC QUINTIC COMPLEX GINZBURG-LANDAU EQUATION

Symbol	Quantity
Ψ	normalized envelope function of the electromagnetic field in dimensionless form
D	group velocity dispersion parameters
γ	higher order correction to the refractive index
ν	higher order correction to the refractive index
δ	linear gain and loss
β	gain dispersion or dissipative term
ε	higher order dissipative contributions
μ	higher order dissipative contributions
x	retarded time
t	propagation distance

TABLE 2. PARAMETERS FOR NONLINEAR CUBIC QUINTIC COMPLEX GINZBURG-LANDAU EQUATION

Symbol	Stationary	Pulsating	Explode	Creep
$D/2$	0.500	0.500	0.500	0.500
γ	1.000	1.000	1.000	1.000
ν	-0.010	-0.100	-0.600	-0.101
δ	-0.100	-0.100	-0.100	-0.100
β	0.500	0.080	0.125	0.101
ε	-0.800	-0.660	-1.000	-1.300
μ	0.100	0.100	0.100	0.300

2.2 Initial Conditions

Fundamental soliton for one-dimensional nonlinear cubic-quintic complex Ginzburg-Landau as refer to Equation (2) presented as

$$\psi(t,x)=\phi(t,x)\exp\left[-i\Omega(t,x)\right] \tag{3}$$

Where the ansatz solution [4] of (2) can be written as

$$\phi(t,x)=\sqrt{\frac{P}{f(t)}}\exp\left[-\frac{x^2}{2r^2f^2(t)}\right], \tag{4}$$

$$\Omega(t,x)=\frac{x^2}{2}\frac{dInf(t)}{dt}, \tag{5}$$

Where P is the peak amplitude at x=0 and $f(t)$ is the variable pulse width parameter so that $rf(t)$ is the width of the soliton. Since we are interested in non-singular bright soliton at t=0, we first look for equilibrium point of the ordinary differential equation, which can be obtained by setting $f(t)$=1 and $(df(t)/dt)$=0. Thus, in the following analysis, we denote r as the width of the soliton.

We have our initial amplitude profile in the form of

$$\psi_0=\sqrt{P}\exp\left[-\frac{x^2}{2r^2}\right], \tag{6}$$

The nonlinear wave phenomena observed in the above mentioned scientific fields, are often modeled by the bell-shaped sech solutions and kink-shaped tanh solutions. The idea originated from the fact that a majority of solitary wave solutions have analytic forms essentially composed of hyperbolic function (cosh, sinh, tanh, cosech, sech ...exp) [5]-[7]. We are able to observe breathing soliton when its convergence of certain initial conditions to a localized solution of the system that is stable for a given set of parameters.

2.3 Modified Equation

It is not easy to solve partial differential equation (PDE) directly. Therefore firstly we keep the equation simple by rearrange one-dimensional cubic-quintic complex Ginzburg-Landau equation (ccqGLE) in (2) to be in this form

$$\frac{\partial\psi(t,x)}{\partial t}=\delta\psi+\left(\beta+i\frac{D}{2}\right)\psi_{xx}-(\varepsilon-i\gamma)|\psi|^{2}\psi-(\mu-i\nu)|\psi|^{4}\psi \tag{7}$$

2.4 Sech Type Initial Amplitude Profile

We set sech type initial amplitude profile for pulsating soliton in the form of

$$\psi(0,x)=\psi_0=\sqrt{P_1}\sec h(x-r_1), \tag{8}$$

Assign value in parameter $P_1=1$ and $r_1=1$, we have

$$\psi(0,x)=\psi_0=\sec h(x), \tag{9}$$

P_1 as the peak amplitude and it can be any integer where $\sqrt{P_1}>0$ while r_1 is the initial width of the initial shape of soliton, but whatever value we assign here it only bring effect at the adjustment interval. Soliton will behave under their self-organization behavior after this adjustment interval.

2.5 Tanh-Sech Type Initial Amplitude Profile

We set tanh-sech type initial amplitude profile for chaotical soliton in the form of

$$\psi(0,x)=\psi_0=\sqrt{P_2}\tan h(x+r_2)-\sqrt{P_3}\tan h(x-r_3)+\sqrt{P_4}\sec h(x-r_4), \tag{10}$$

Assign value in parameter $P_2=1$, $P3=1$, $P_4=0.4$ and $r_2=10$, $r_3=10$, $r_4=0$, we have

$$\psi(0,x)=\psi_0=\tan h(x+10)-\tan h(x-10)+0.4\sec h(x), \tag{11}$$

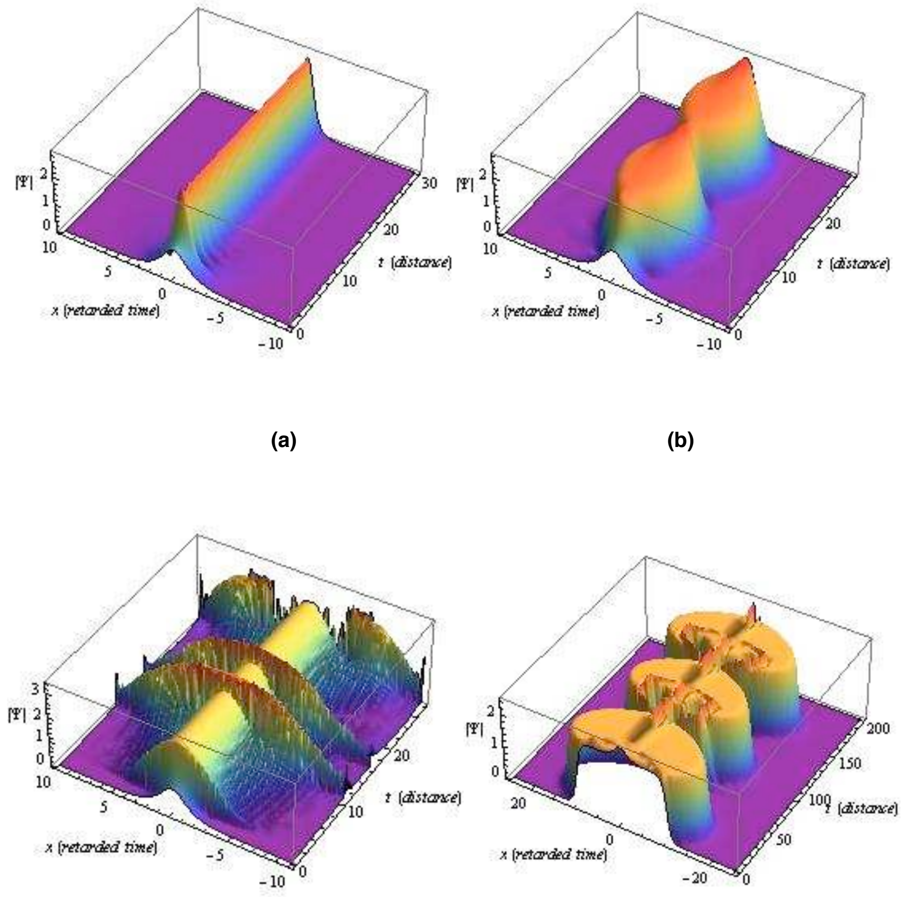
P_n as the peak amplitude and it can be any integer where $\sqrt{P_n}>0$, for $n=2,3,4$ while r_m is the initial width of the initial shape of soliton where $m=2,3,4$. But whatever value we assign here it only bring effect at the adjustment interval. Soliton will behave under their self-organization behavior after this adjustment interval.

3 Result of Numerical Simulation on Breathing Solitons

The formation can move with constant velocity when the matter supply is too scarce for stationary but when stationary state does not provides a sufficient rate of energy supply, our dissipative solitons can oscillate and give adjustment pattern of solitary waves solutions based on self-organization concept; pulsating, exploding and creeping solitons. But the structure can disappears when the source is switched off or if the parameters are moved outside the range of existence of the soliton solutions [8].

They do not transform to each other unless we disturb the soliton by changing the parameters of the system to the extent that one of the solitons becomes unstable and cause opposite sides of the breathing solution to breathe at different frequencies.

The stationary soliton as fundamental solution of cqCGLE and its soliton able to maintains good stability during its propagation. Pulsating soliton or limit cycle appear due to the dynamic balance between dissipation and energy supply. The soliton width varies periodically, while the soliton amplitude is closer to a constant in each case. Exploding soliton evolution starts from a stationary localized solution that has a prefect soliton shape. After a while, its slopes become covered with small ripples that seem to move downwards along the two slopes of the soliton and very soon the pulse is covered with this seemingly chaotic structure. When the ripples increase in size, the soliton cracks into pieces like an explosion, then it will back to the perfect soliton shape. Creeping soliton is solitons that changes its shape periodically from a bell shaped pulse to a long flat-top profiles that consist of two fronts at the sides of the soliton and shifts a finite distance in the transverse direction after each period of oscillation, thus creating creeping movements of the whole "worm-like" formation.



(a) (b)

(c) (d)

Figure 2. Result of breathing pattern of (a) Stationary soliton, (b) Pulsating soliton, (c) Exploding soliton and (d) Creeping soliton.

4 DISCUSSION

Dissipative solitons can oscillate or change it shape due to loss and gain of energy over long propagation allows us to observe the breathing pattern during its propagation. They do not transform to each other unless we change the parameters of the system that represent the environment changes that present loss and gain of energy along the propagation. By understanding this special behaviour, make the solitons suitable to explain the mechanism in optical fiber and nerve signals in transmit information or data of undistorted signals. Even when loss is present, the soliton changes its shapes as it has to lose energy while travelling, but it still maintains spatial localization of wave energy in the changing pulse shape through a unique solitons; dissipative solitons not breaking into multiple sinusoids.

5 CONCLUSION

This paper shows pattern of breathing solitons along its propagation in numerically by assigning hyperbolic sine and hyperbolic tangent initial amplitude profile in wave equation; cubic-quintic complex Ginzburg-Landau. Balance effect between nonlinear and dispersion in wave equation give the ability to the wave to propagate in long distance, while the effect of loss and gain of energy represent the interaction between pulses and its environment changes when we change the parameters along the propagation. As the energy supply finish, a dissipative soliton ceases to exist, thus energy supply must be continuous and balance between it loss and gain.

6 ACKNOWLEDGMENT

We thank the referees for their valuable comments. We acknowledge support from School of Mathematical Sciences, Universiti Sains Malaysia which enabled us to carry out this work.

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