A REVIEW ON THE PERFORMANCE OF MODIFIED CAM CLAY MODEL IN PREDICTING THE MECHANICAL BEHAVIOUR OF HEAVILY OVERCONSOLIDATED CLAY

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Abstract

The Modified Cam Clay (MCC) is one of the most commonly used soil models, which is very popular in the world. It was developed by the researchers at the University of Cambridge, U.K. This MCC model has gained general acceptance amongst the researchers in the field of geotechnical engineering. Some commercial softwares for geotechnical problems are using the MCC model as the basis of analysis. The MCC model undoubtedly works very well for predicting the mechanical behavior of normally consolidated clay, but in reality some soils are in the state of overconsolidated. This study is aimed to find out some pitfalls in using the MCC model in relation to its application on overconsolidated soils. Some data from plane strain testing on the overconsolidated clay were used to examine the application of MCC model on overconsolidated clay. The results showed that, the MCC model is no longer able to simulate the mechanical behavior of heavily overconsolidated clay; the MCC calculation for heavily overconsolidated clay was deviated far below the experimental result.

Keywords:
Modified Cam Clay, normally consolidated clay, overconsolidated clay, plane strain testing.

INTRODUCTION

The Modified Cam Clay (MCC) is one of the most reliable constitutive soil models in the world. This is indicated by well acceptance shown by the great majority of the researchers in the field of geotechnical engineering. The MCC model is able to simulate the mechanical behavior of normally consolidated clay satisfactorily.

The question then might arise when the MCC model is going to apply for one particular state of soil, which is called as overconsolidated soil. One of the problems, which come forward, is the stress history of the overconsolidated soil, which is different to the normally consolidated clay.

This study discusses the application of the MCC model for replicating the mechanical behavior of overconsolidated soil. This is to find some pitfalls in the use of the MCC model in dealing with the oc-clay. Several data from plane strain testing on overconsolidated soil were used to verify the accuracy of the MCC model.

Constitutive model/law

The purpose of a constitutive law is to present a mathematical model that describes the behavior of a material. The physical behavior that has been perceived mentally can be modeled by a constitutive law. The ultimate target of using a mathematical model is to gain the ideas to solve events quantitatively. Furthermore, the reliability of a constitutive model is strongly depending upon the extent to which the physical phenomenon has been understood and simulated.

Study of the response of a substance or body, which subjected to external loading or excitation, creates the major endeavor in engineering and sciences. The important factors in such study are: (1) external excitation (2) internal constitution of the medium, and (3) the response. It has been observed that materials with the same geometry but with different internal constitution respond in different ways to the same external excitation.
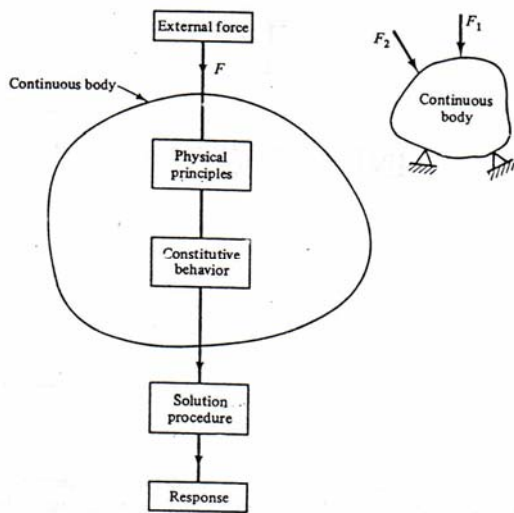
Figure 1. Role of constitutive law in continuum mechanics (Desai and Siriwardane, 1984)

Consider two different materials, which are made from different material but with the same geometry, say rubber and steel bricks. If such materials subjected to the same load, the response of these two materials will be different, the rubber brick will deform more than steel brick. The load-deformation behaviour or in general the cause-and-effect relation will depend upon how the material is constituted. The internal constitution of a material affects the deformation behaviour of the body. This constitution of material governs the behaviour of the body. The relationship between cause and effect can be called as the constitutive law of the material for a given phenomenon (Desai and Siriwardane, 1984).

Modified Cam Clay Model
The yielding of soils started to be studied by the emergences of Rendulic (1936) and Hvorslev (1960) works. Subsequently, Roscoe and his co-workers (1958, 1963 and 1962) at Cambridge University investigated the findings of Rendulic and Hvorslev by proposing models for yielding of soils, which was based on the theory of plasticity. The most important parameters that used in the critical state models are *p (mean stress)*, *q(deviatoric stress)* and *e(void ratio)*. Related to the triaxial configuration, these parameters take the forms described below.

Derivation of the Modified Cam Clay Model
This following exposition of the Modified Cam Clay model is adopted from Desai and Siriwardane (1984).

For axisymmetric triaxial conditions, $\sigma_2 = \sigma_3$ and $\varepsilon_2 = \varepsilon_3$, hence the work which is acting on the

specimen per unit volume can be expressed by the following equation:

$$dW = \sigma_1 d\varepsilon_1 + 2d\varepsilon_3$$

...........[1]

$$= \left(\frac{\sigma_1 + 2\sigma_3}{3}\right)(d\varepsilon_1 + 2d\varepsilon_3) + (\sigma_1 - \sigma_3)\frac{2}{3}(d\varepsilon_1 - d\varepsilon_3)$$

dW can also be expressed by :

$$dW = pd\varepsilon_v - qd\varepsilon_s \ ....[2]$$

Where:
$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

$$d\varepsilon_v = d\varepsilon_1 + 2d\varepsilon_3$$

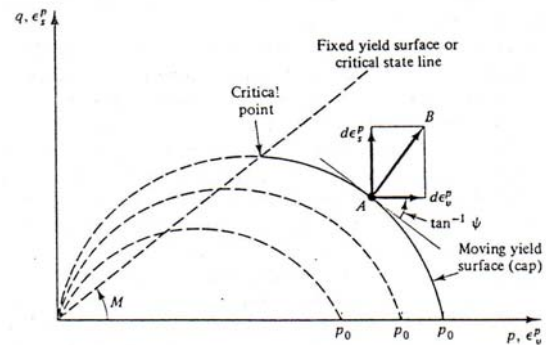$$d\varepsilon_s = \frac{2}{3}(d\varepsilon_1 - d\varepsilon_3)$$

Figure 2. Yield locus in *q-p* space

Figure 2 depicts a projection of the critical state line on the *q-p* space with projections of typical sections of the state boundary surface. The projection of the state boundary surface form continuous curves and is referred to as yield surfaces, yield loci or yield caps.
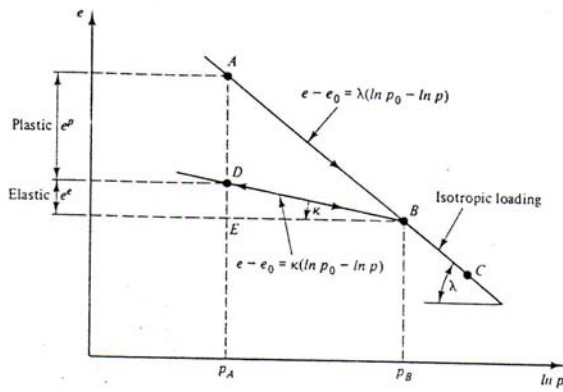
Figure 3. Consolidation behaviour in e-ln p space

It can be inferred from Figure 3:

$$e = e_A - e_B = \lambda\left(\ln p_B - \ln p_A\right)$$
$$e^e = e_D - e_E = \kappa\left(\ln p_B - \ln p_A\right)$$ .... [3]

By differentiating both equations (3) above resulted:

$$de = -\lambda\frac{dp}{p}$$ .... [4a]

$$de^e = -\kappa\frac{dp}{p}$$ .... [4b]

Where superscript e denotes the recoverable elastic component. Hence:

$$d\varepsilon^p = de - de^e = -\left(\lambda - \kappa\right)\frac{dp}{p}$$ .... [5]

Where de^p is the plastic component of the incremental void ratio.

With compressive volumetric strain positive, the following equation is resulted:

$$d\varepsilon_v = -\frac{de}{1+e_0} = \frac{\lambda}{1+e_0}\frac{dp}{p}$$ .... [6]

Elastic volumetric strain, $\delta\varepsilon_v^e$:

$$d\varepsilon_v^e = -\frac{de^e}{1+e} = \frac{\kappa}{1+e}\frac{dp}{p}$$ .... [7]

It is assumed in the critical state theory that there is no recoverable energy associated with shear distortion. Therefore:

$$d\varepsilon_s = d\varepsilon_s^p$$ .... [8]

According to Figure 2:

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = -\frac{dp}{dq}$$ .... [9]

Where $d\varepsilon_v^p$ is the volumetric plastic strain.

Equation of yield locus

Stress ratio can be defined as:

$$\eta = \frac{q}{p}$$ .... [10a]

or

$$q = \eta p$$ .... [10b]

Therefore

$$dq = pd\eta + \eta dp$$ .... [11]

It is assumed that the slope of the yield curve at any point (p, q) as in Figure 2, be ψ. Since q decreases with p, the sign of ψ is negative:

$$\frac{dq}{dp} = -\psi$$ .... [12a]

or

$$dq = -\psi dp$$ .... [12b]

Substitution equation (11) in equation (12a) will yield:

$$pd\eta + \eta dp = -\psi dp$$ .... [13]

Equation (13) can also be expressed as:

$$\frac{dp}{p} + \frac{d\eta}{\eta + \psi} = 0$$ .... [14]

Equation [14] actually expresses the yield locus. Since the model assumed that the successive yield loci are geometrically similar, ψ is a function of η only. Any yield curve passing through a known point can be obtained by integrating equation (14):

$$\int_{p_0}^{p}\frac{dp}{p} + \int_{0}^{\eta}\frac{d\eta}{\eta + \psi} = 0$$ .... [15a]

$$\ln p - \ln p_0 + \int_{0}^{\eta}\frac{d\eta}{\eta + \psi} = 0$$ .... [15b]

Equation [15b] represents the yield curve passing through (p_0, 0), here p_0 is treated as a variable which has a unique value for any yield surface. In fact this can be considered as hardening parameter. Equations [15] can be expressed in the differential form as:

$$\frac{dp_0}{p_0} - \frac{dp}{p} - \frac{d\eta}{\psi + \eta} = 0 \quad \text{......................................} [16]$$

When the material changes its state from one yield locus to another, the change in hardening parameter is the same irrespective of the stress path followed.

$$de = -\lambda \frac{dp_0}{p_0} \quad \text{..} [17a]$$

$$de^e = -\kappa \frac{dp_0}{p_0} \quad \text{..} [17b]$$

Hence:

$$de^p = -(\lambda - \kappa)\frac{dp_0}{p_0} \quad \text{..} [17c]$$

Substituting equation (16) to equation (17c) we can obtain:

$$de^p = -(\lambda - \kappa)\left(\frac{dp}{p} + \frac{d\eta}{\psi + \eta}\right) \quad \text{..........................} [17d]$$

Therefore, the plastic volumetric strain can be written as:

$$d\varepsilon_v^p = \frac{de^p}{1+e} = \frac{\lambda - \kappa}{1+e}\left(\frac{dp}{p} + \frac{d\eta}{\psi + \eta}\right) \quad \text{.....................} [18]$$

The ratio ψ of plastic components of shear and volumetric strain can be obtained by considering the dissipated energy while undergoing deformation on the state boundary surface. It is assumed in MCC model that:

$$dW = Mpd\varepsilon_s \quad \text{..} [19]$$

Hence

$$pd\varepsilon_v^p + qd\varepsilon_s^p = Mpd\varepsilon_s \quad \text{....................................} [20a]$$

By assuming that $d\varepsilon_s = d\varepsilon^p$, the following equation is resulted:

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = \frac{1}{M - \eta} \quad \text{..} [20b]$$

In the model, the associated flow rule is assumed, hence the incremental strain vector AB in Figure 2 is normal to the yield surface, consequently the ratio of $\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = \frac{1}{\psi}$ which leads to:

$$\psi_c = M - \eta \quad \text{..} [21]$$

Where subscript c denotes ψ for Cam Clay model.

In MCC model, the dissipated energy dW is assumed as:

$$dW = p\sqrt{\left(d\varepsilon_v^p\right)^2 + M^2\left(d\varepsilon_s^p\right)^2} \quad \text{..........................} [22]$$

This leads to:

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = \frac{2\eta}{M^2 - \eta^2} \quad \text{...} [23a]$$

Therefore:

$$\psi_{cm} = \frac{M^2 - \eta^2}{2\eta} \quad \text{..} [23b]$$

Here the subscript c_m denotes modified Cam Clay model. Once ψ known, the yield locus for the modified Cam Clay model can be found by integrating the following equation:

$$\int_0^\eta \frac{2\eta d\eta}{M^2 + \eta^2} = -\int_{p_0}^p \frac{dp}{p} \quad \text{.....................................} [24a]$$

$$\ln\left(M^2 + \eta^2\right) - \ln\left(M^2\right) = -\ln p + \ln p_0 \quad \text{...............} [24b]$$

Simplification, leads to:

$$\frac{M^2 + \eta^2}{M^2} = \frac{p_0}{p} \quad \text{..} [24c]$$

Or

$$M^2 p^2 - M^2 p_0 p + q^2 = 0 \quad \text{................................} [24d]$$

This is for the equation of an ellipse on q-p plot. Substituting the value of ψ_{cm}, one can write the

following expressions for incremental quantities based on Modified Cam Clay model:

$$d\varepsilon_v^p = \frac{\lambda - \kappa}{1+e}\left(\frac{dp}{p} + \frac{2\eta d\eta}{M^2 + \eta^2}\right) \quad\text{........................ [25a]}$$

$$d\varepsilon_v = \frac{\lambda}{1+e}\left[\frac{dp}{p} + \left(1 - \frac{\kappa}{\lambda}\right)\frac{2\eta d\eta}{M^2 + \eta^2}\right] \quad\text{................ [25b]}$$

$$d\varepsilon_s^p = \frac{\lambda - \kappa}{1+e}\left(\frac{dp}{p} + \frac{2\eta d\eta}{M^2 + \eta^2}\right)\frac{2\eta}{M^2 - \eta^2} \quad\text{........... [25c]}$$

Behaviour of overconsolidated clays

Overconsolidated clays (oc-clays) have different behaviour to normally consolidated clays (nc-clays). Both types of clays have different engineering characteristics due to their stress histories that they underwent in the past. The state of nc-clay lies on the "wet" side of the critical state line while the state of oc-clay is on the "dry" side of critical state line. Figure 4 shows the failure states of overconsolidated clays, which fall on the Hvorslev surface. The detailed discussion of oc-clay behaviour is discussed in Atkinson and Bransby (1982).
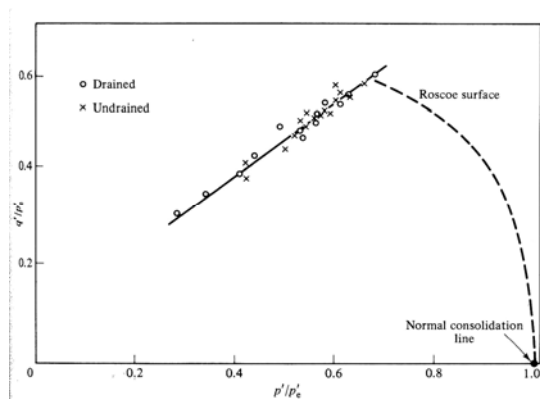
Figure 4. Failures of overconsolidated soils on Hvorslev surface (Atkinson and Bransby, 1982)

METHOD
Plane strain compression Tests

To study the behaviour of overconsolidated clays, a series of biaxial compression or plane strain compression tests was undertaken. The clay that used for this study was obtained from UNIMIN PTY.LTD, Australia, with the commercial name Kaolin RF. This Kaolin RF was firstly mixed with water in such a way so that its water content was 1.5 times its liquid limit. This mixture was then poured into a cylindrical mould as seen in Figure 5, to be pressurized to form the specimen for biaxial tests.

Figure 5. Consolidation of kaolin slurry in cylindrical mould

Before running the compression tests in the biaxial apparatus, the specimen was trimmed so that fitted to the apparatus, the specimen size was 72 mm x 72 mm x 36 mm. Standard procedure for either CD or CU triaxial testing was applied (Head, 2002). A backpressure of 300 kPa was applied for every test, to achieve fully saturated specimen, confining pressure was applied, which was about 10 kPa higher than backpressure. The saturation stage was fully achieved by checking the B-value, which was around 0.96-0.98. Once the specimen reached its saturation condition, the specimen was then treated to experience loading-unloading stages; the maximum effective consolidation pressure of 700 kPa was applied for all the test specimens. The unloading process was undertaken to set the test specimen in overconsolidated condition, by reducing the confining pressure using several steps until reaching the required overconsolidated ratio (OCR). For example, if the desired OCR is 4, the isotropic swelling process was carried out in 3 steps of unloading process, that are 700 kPa, 350 kPa and 175 kPa. The test specimen for drained condition (CD) is loaded axially by applying the loading through the loading machine, with the drainage line opened, which was connected to volume change measurement device. The drained test needed around 3 days to complete, data were read every 5 minutes. For CD tests, the loading rate of 0.004 mm/min was applied. This loading rate was deduced based on the drainage condition of the adopted kaolin clay (Bishop and Henkel, 1962). The drained compression test was terminated at the axial strain of about 20% or sooner. Once the test specimen had failed, it was taken out immediately for the purpose of moisture content determination. Initial treatment for CU tests were carried out using the same methods as applied to CD test specimen, the difference is just how it is sheared to failure. The loading rate for CU tests was 0.008 mm/min, with the drainage line was closed; pore pressure was measured during the application of axial load.

Test plan for this study is as shown in Table 1.

Table 1. Test plan of plane strain compression tests on Kaolin RF clay

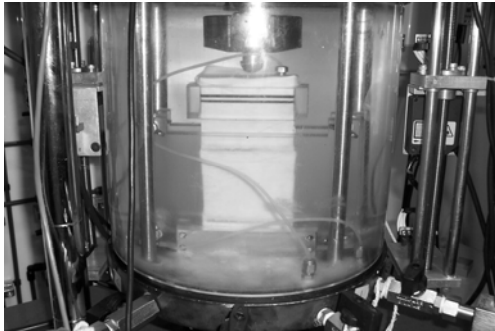| Series	Test type & OCR
PSC700	Undrained, 1
PSCU4	Undrained, 4
PSCU16	Undrained, 16
PSCD8	Drained, 8
PSCD20	Drained, 20

Figure 6. Complete set up of plane strain testing

Analytical Method

The analytical method is aimed to compute the values of mechanical properties of the overconsolidated clay being considered based on Modified Cam Clay framework. The calculation would follow these several steps below:

Step1:
Select values of mean pressure, p and compute the corresponding value of q, it is also possible to select q then calculate p.

Step 2:
Compute the value of stress ratio, $\eta = q/p$

Step 3:
Compute increment in stress ratio, η_{inc} as

$$\eta_{inc} = \left(\frac{q}{p}\right)_i - \left(\frac{q}{p}\right)_{i-1}$$

Where i denotes as incremental step.

Step 4:
Compute the value of $d\varepsilon_v$ during increment of stress using:

$$d\varepsilon_v = \frac{\lambda}{1+e}\left[\frac{dp}{p} + \left(1 - \frac{\kappa}{\lambda}\right)\frac{2\eta d\eta}{M^2 + \eta^2}\right]$$

Step 5:
Find the total volumetric strains by accumulating incremental quantity computed in step 4.

Step 6:
Compute the change in void ratio from:

$$\frac{de}{1+e} = d\varepsilon_v$$

Step 7:
Compute the current void ratio after current load:

$$e = e - de$$

Step 8:
Compute the incremental shear strain using :

$$d\varepsilon_s = \frac{\lambda - \kappa}{1+e}\left(\frac{dp}{p} + \frac{2\eta d\eta}{M^2 + \eta^2}\right)\frac{2\eta}{M^2 - \eta^2} = d\varepsilon_v^p \frac{2\eta}{M^2 - \eta^2}$$

The total shear strain (ε_s) can be computed by accumulating the incremental quantities.

Table 2   Properties of adopted Kaolin RF clay

Parameters	Values
λ, virgin compression line slope	0.103
κ, swelling line slope	0.04
N, specific volume at $p'=1$ kPa (NCL)	2.10
Γ, specific volume at $p'=1$ kPa (CSL)	2.04
M, critical state line slope	0.93
h, Hvorslev surface slope	0.85
α, Hvorslev surface intercept, kPa	27.85
p_c', effective pre-consolidation pressure, kPa	700
G_s, specific gravity	2.6
LL (%)	53.5
PL (%)	30.8
PI (%)	22.7

RESULTS AND DISCUSSION
Numerical Analysis of a Test Problem

A set of triaxial testing data on clay specimens obtained from Desai and Siriwardane (1984) was used to be simulated using Modified Cam Clay. The material parameter are : M = 1.0, λ = 0.174, κ = 0.026, e_0 = 0.889. The specimen which was used for this example was initially consolidated to p_0'=30 psi and then was sheared until failure under drained condition.

Figure 7 shows the drained stress-strain curves, which are generated, from either MCC prediction and experimental result. It is clearly seen that the MCC model is able to predict the behaviour of real triaxial testing satisfactorily, which is shown by the stress-strain curve that it has which is very close to the experimental result.
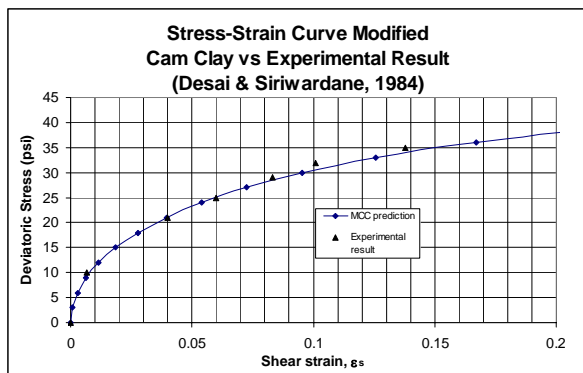
Figure 7. Comparison between MCC prediction and experimental result (Desai & Siriwardane, 1984)

Plane strain compression tests on NC-Clay

In addition to running the plane strain testing on overconsolidated Kaolin RF clay, testing on normally consolidated clay (nc-clay) was also carried out using the same device. The specimen was isotropically consolidated up to 700 kPa and then it was loaded until failure under undrained condition. The experimental stress-strain curve of this nc-clay is depicted in Figure 8. A back calculation using MCC model was undertaken to simulate this experimental data and the calculation result was plotted in Figure 8 as well. It can be seen clearly that the MCC model was able to predict the behavior of normally consolidated clay quite accurately (see Figure 8).
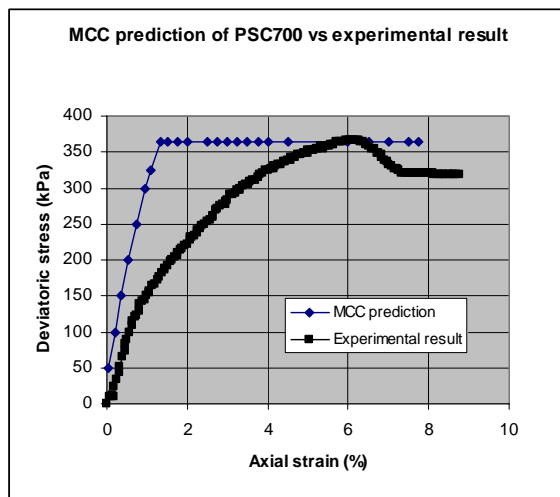
Figure 8. Comparison between the MCC calculation vs experimental result of nc-clay

Figure 9 shows the comparison between the stress paths that are resulted from the MCC calculation versus the experimental result. The MCC effective stress path was in good agreement with the experimental result; both stress paths fell on the critical state line with the similar path pattern. This

fact confirms that the MCC model is quite good in simulating the mechanical behavior of nc-clay.
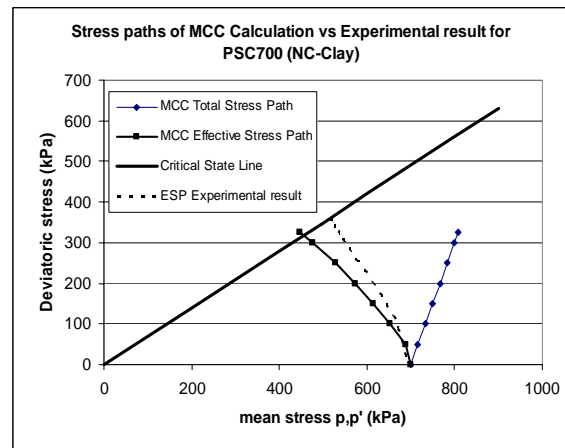

Figure 9. Stress paths of the MCC and experimental result

Comparison between the MCC simulation and the oc-clay test results

Figure 10 and 11 show the results of biaxial compression tests on overconsolidated clays. It can be seen from the stress-strain curves that, the specimens reached their peak stresses at the strains of around 2.5-3%. After this peak stress, it is followed by the strain softening mechanism in which the axial strain would increase without the increase of deviatoric stress. This is the typical behavior of overconsolidated soil when it is sheared. It can be found from observation carried out in the laboratory during compression test, the specimens in general formed shear bands upon reaching the peak stress, this shear band leads to bifurcation of the specimens into two blocks. After shear bands appearance, the specimens relied their strengths on the shear, which occurred between the two blocks of the specimens, which moved towards the opposite direction.

The comparison between the MCC calculations and the experimental results were depicted in Figure 11 and 12. It can be seen clearly that the MCC model was unable to simulate the mechanical behaviour either for the specimen with the OCR of 8 and OCR of 20, the MCC model calculation was deviated quite far from the actual behaviour of oc-clay. The trace of consolidation process that is undergone by the oc-clay could not be replicated by the MCC model, in which the extra strength of the oc-clay is gained due to higher pre-consolidation stress compared to the current stress acting on the soil specimen. It can be said that the use of the MCC model to replicate the mechanical behaviour of oc-clay will lead to the conservative calculation.
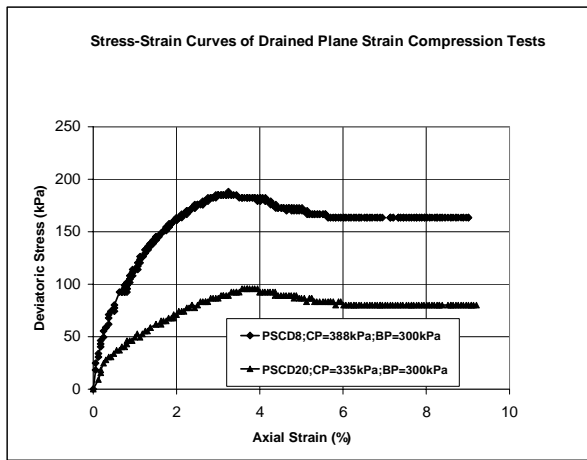
Figure 10. Stress-Strain Curves of Drained Plane Strain Compression Test on Overconsolidated Clays
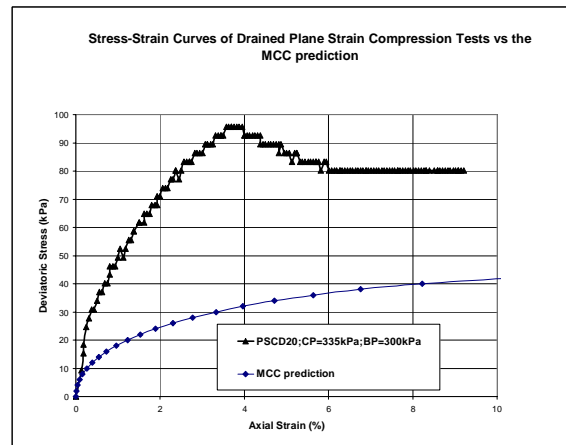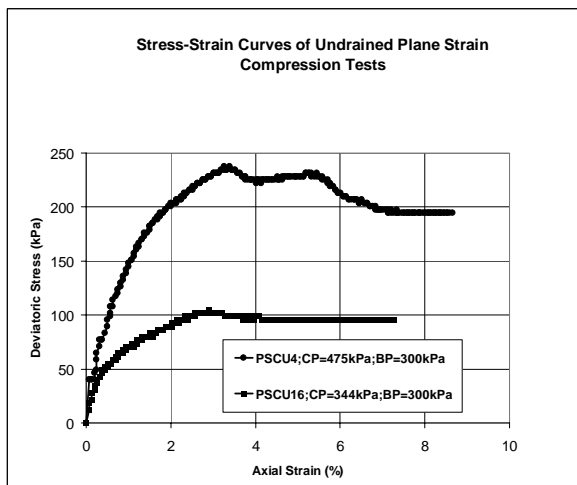

Figure 11. Stress-Strain Curves of Undrained Plane Strain Compression Test on Overconsolidated Clays

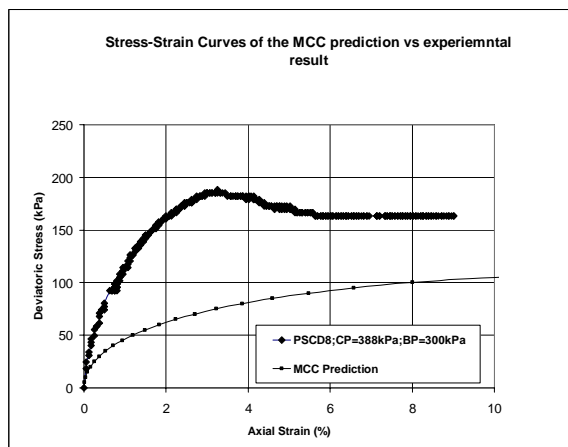Figure 12. Comparison between the MCC calculation and experimental result (PSCD8)

Figure 13. Comparison between the MCC calculation and experimental result (PSCD20)

CONCLUSION

The comparison between the MCC model calculation and the experimental results on oc-clay has been made. Several conclusions can be drawn from this study are as follows:

1. The MCC model is able to simulate the mechanical behavior of normally consolidated clay satisfactorily. While it fails to predict the mechanical behavior of heavily overconsolidated clay.

2. The use of the MCC model for simulating the mechanical behavior of heavily overconsolidated clay will lead to a conservative design which will result much lower strength compared to the actual strength of the heavily overconsolidated clay.

ACKNOWLEDGMENT

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