

## AN ECONOMIC ANALYSIS OF VARIABLE RATE TECHNOLOGY

Isti Surjandari <sup>1</sup>, Marvin T. Batte <sup>2</sup>, and Mario J. Miranda <sup>2</sup>

<sup>1</sup> Industrial Engineering Department, Faculty of Engineering, University of Indonesia, Depok 16424, Indonesia

<sup>2</sup> AED Economics, The Ohio State University, OH 43210, USA

*E-mail: isti@ie.ui.ac.id*

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### Abstract

Variable Rate Technology (VRT) offers an opportunity to improve production efficiency by allowing input applications to fluctuate in response to spatial variations in soil characteristics and nutrient levels. Society may also benefit from reduced negative externalities, such as surface and groundwater contamination, from input applications. Using a dynamic spatial model, this study examines how the interaction among variability, spatial autocorrelation, and mean level of soil fertility affects optimal sampling density and the economic gains from VRT. VRT was found to be profitable under selected conditions, and the optimal grid size will vary with these conditions. In the case where variability and mean fertility levels are significantly high associated with low spatial autocorrelation, VRT produces greater net returns than Uniform Rate Technology (URT), even with the smallest grid size to base the input application decisions. Results also demonstrate that optimal grid size increases with increased spatial autocorrelation.

*Keywords: spatial autocorrelation, spatial variability, variable rate technology.*

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### 1. Introduction

Global Positioning Systems (GPS) and Geographical Information Systems (GIS) are two key technologies that enable the emergence of Site-Specific Management (SSM) technology. While GPS allows producers to identify field locations so that inputs can be applied appropriately to individual field locations, GIS technology allows users to store field input and output data as separate layers in a digital map and to retrieve and utilize these data for future input allocation decisions [1]. With the availability of supporting technologies, SSM allows producers to (1) capture detailed field spatial data, (2) interpret and analyze that data, and (3) implement an appropriate management response based on the information.

Variable Rate Technology (VRT) is a central component of the much-touted Site-Specific Technologies. VRT allows producers to allocate inputs more efficiently by exploiting spatial variations in soil type, fertility levels, and other field characteristics. A potential consequence of VRT for the producer is greater profit arising from higher yield and/or reduced variable input costs. Society may also benefit from reduced negative externalities such as surface and groundwater contamination from fertilizers and pesticides.

Variation in soil attributes is a necessary condition for the profitable application of VRT. That is, if a field is homogenous, then the optimal input application will not vary across the field and VRT creates no value. If the field is not homogenous, however, VRT has potential value provided that appropriate input recommendations for each part of the field can be derived.

Accurate estimation of field characteristics is key to the successful implementation of VRT. Increased sampling density allows the input application to be better tailored to the individual site characteristics. However, increased sampling also comes at higher cost. Grid Soil Sampling (GSS) involves dividing a field into square sections of certain acreage and gathering soil samples from each section. With the aid of DGPS as a positioning system, producers can identify the location of each grid. In general, the optimal soil sampling density depends on a number of factors, including: (1) the smallest possible area that can be managed under current technology, (2) the expected variability in soil fertility, and (3) costs of soil sampling.

Another factor that affects optimum grid sampling density is the spatial autocorrelation of soils and other field characteristics, and ultimately, crop yields. A positive spatial autocorrelation is commonly found in agronomic studies, where neighboring areas are more similar than those farther apart. Similarity in soil type

and other field characteristics in neighboring areas may cause those areas to have the same yield-limiting factors. For the same variability in soil fertility, the more spatially correlated the soil nutrients, the larger the economically optimum grid size will be. In other words, if neighboring areas have similar soil and field characteristics, then aggregation errors will be relatively small and a small grid size may not be warranted.

A number of studies have examined the economic feasibility of VRT. These studies have produced mixed findings, leaving the profitability issue unresolved. This divergence of findings may result from the use of different rules to allocate the variable resource or from differences in the mean level or variability of nutrients within the field. Differences in spatial autocorrelation of field characteristics also may partially explain the contradictory results. Indeed, it is quite likely that all of the above conditions combine to explain the differences observed. This study will address the inconsistent results in the literature by jointly considering the impact of level, variability, and spatial autocorrelation of soil nutrients within a field on the economic performance of VRT fertilizer applications.

The primary objective of this study is to examine the economic gains from VRT Phosphorus (P) and Potassium (K) applications in a corn-soybean rotation. Alternative fertility distributions with different variability and spatial autocorrelation profiles will be generated to analyze their effect on optimum grid size and the economic performance of VRT. A dynamic spatial model will be used to capture the temporal and spatial variations in soil nutrients. The dynamic features of the model allow us to capture the carry-over process of soil nutrients; the spatial features of the model allow us to capture interconnections of soil nutrients among neighboring grids.

## 2. Methods

Spatial autocorrelation of the data may exist if observed values display a non-random pattern in space. While variability in soil fertility is commonly expressed in terms of the coefficient of variation (CV), spatial autocorrelation is generally expressed in terms of a correlation coefficient, such as the Moran statistic, which indicates the degree of similarity between association in value and association in space. The Moran ( $I'$ ) statistic can be written as [2]:

$$I' = \frac{n \sum_{i,j} (x_i - \bar{x})(x_j - \bar{x}) w_{ij}}{\sum_{i,j} w_{i,j} \sum_i (x_i - \bar{x})^2} \quad (1)$$

where  $n$  is the number of zones,  $x_i$  is the value of variable  $x$  in zone  $i$ , and  $w_{ij}$  is the weight. The Moran statistic can take on values between  $-1.0$  and  $+1.0$ . The weights usually take the form of a binary contiguity matrix, where the elements of the  $w_{ij}$  matrix take on values of 1 if observations  $i$  and  $j$  are neighbors, and zero values otherwise [3-4].

This study uses dynamic simulation with hypothetical data to examine how the interaction among variability, spatial autocorrelation, and mean level of soil fertility affects optimal sampling density and the economic gains from VRT. The soil P and K values generated for the hypothetical field will display alternative mean levels, variability and spatial autocorrelation of soil fertility. The simulated soil fertility data will be treated as 'truth'. The performance of VRT of P and K, assuming different grid sizes, will be examined in this hypothetical universe.

For the purpose of this study, a hypothetical 90-acre field of corn-soybean rotation under northwest Ohio conditions will be used as the basis of analysis. The hypothetical field will be divided into 144 grids with an area of 0.625-acre each in the base scenario. In alternate scenarios, larger grid sizes of 2.50, 5.625, 10, 22.50, and 90 acres are considered. The 90-acre grid corresponds to one management zone per field, and thus represents the case of Uniform Rate Application. Soil P and K values for each of these grids is obtained by taking the average of the soil test values from the base model (i.e., soil P and K values from the 0.625-acre grid size). Thus, soil P and K values for each of the larger grids is given by:

$$S = \frac{1}{g} \sum_{i=1}^g S_i \quad (2)$$

where  $S_i$  is the 'true' nutrient level for each 0.625-acre grid contained in the larger grids,  $S$  is the blended (average) nutrient level for the larger grid, and  $g$  is the number of small cells within each larger grid. Empirical soil test data for P and K in northwest Ohio is used as guidance in the generation of soil P and K values.

A multivariate lognormal distribution of soil nutrient values with correlation coefficient of 0.40 is generated for the 0.625-acre grid. Different soil fertility distributions will be generated by varying three parameter values: (1) the mean of the soil nutrient values, (2) the CV of the soil nutrient values that captures the degree of variability, and (3) the spatial autocorrelation coefficient that capture the association in value and space of the soil nutrients. Two values are assumed for each of these three parameter values. Additionally, a high mean fertility scenario is included, resulting in 20 fertility distribution scenarios. Parameter

values for coefficient of variation are 30 and 80 percent, and the spatial autocorrelation coefficients (Moran statistic) are 0.25 and 0.75.

In this study, spatial autocorrelation is represented by the Moran statistic, which is expressed in equation (1). A first order binary contiguity matrix based on the Queen criterion is used for the weight. Using a simulation algorithm developed by Goodchild [5], a fertility distribution with a specific Moran coefficient can be obtained by doing a spatial rearrangement of the soil nutrient values.

Specifications for the production and carry-over functions are obtained from Schnitkey et al. [6]. The production function has the following form:

$$Y_{i,j} = \alpha_{0,j} \left[ \frac{1 - 10^{(-\alpha_{1,j} X_{i,P} - \alpha_{2,j} S_{i,P,t})}}{1 - 10^{(-\alpha_{3,j} X_{i,K} - \alpha_{4,j} S_{i,K,t})}} \right] \quad (3)$$

where  $Y_{i,j}$  is crop yield in bushels per acre from grid  $i$  for crop  $j$  ( $j$  = corn when  $t$  is even and soybeans otherwise),  $X_{i,P}$  and  $X_{i,K}$  represent the amount of P and K applied per acre,  $S_{i,P,t}$  and  $S_{i,K,t}$  represent the amount of P and K per acre in the soil at the beginning of growing season  $t$ ,  $\alpha_j$  and  $\beta_j$  are parameters of the production function. For corn,  $\alpha_{0,j} = 164$ ,  $\alpha_{1,j} = 0.0091$ ,  $\alpha_{2,j} = 0.043$ ,  $\alpha_{3,j} = 0.008$ , and  $\alpha_{4,j} = 0.0064$ . For soybeans,  $\alpha_{0,j} = 45$ ,  $\alpha_{1,j} = 0.0071$ ,  $\alpha_{2,j} = 0.0540$ ,  $\alpha_{3,j} = 0.157$ , and  $\alpha_{4,j} = 0.0038$ .

The carry-over function is specified as follow:

$$S_{i,f,t+1} = S_{i,f,t} + \beta_{f,j,1} X_{i,f} - \beta_{f,j,2} Y_{i,j} - \beta_{f,j,3} \dots \dots \dots (4)$$

where  $S_{i,f,t+1}$  is the amount of soil nutrient  $f$  ( $f$  = P, K) at the beginning of growing season  $t+1$  in grid  $i$ , and  $\beta_{f,j}$  are carry-over function parameters. For P carry-over in corn,  $\beta_{f,j,1} = 0.1$ ,  $\beta_{f,j,2} = 0.037$ ,  $\beta_{f,j,3} = 0.0$ . For P carry-over in soybeans,  $\beta_{f,j,1} = 0.1$ ,  $\beta_{f,j,2} = 0.08$ ,  $\beta_{f,j,3} = 0.0$ . For K carry-over in corn,  $\beta_{f,j,1} = 0.25$ ,  $\beta_{f,j,2} = 0.0675$ ,  $\beta_{f,j,3} = 5.0$ . For K carry-over in soybean,  $\beta_{f,j,1} = 0.25$ ,  $\beta_{f,j,2} = 0.35$ ,  $\beta_{f,j,3} = 5.0$ .

With VRT, input applications for each grid depend only on that grid's soil nutrient values. Therefore, the optimal rates for each grid are found independently of other grids. By assuming that the producer wishes to maximize expected profits, the optimum rates of P and K are obtained using dynamic programming, that is, by solving the following Bellman equation:

$$V_i(S_{i,P,t}, S_{i,K,t}) = \max_{X_{i,P}, X_{i,K}} (p_j \cdot f_j(\cdot) - w_P X_{i,P,t} - w_K X_{i,K,t}) + \delta V_i(S_{i,P,t+1}, S_{i,K,t+1}) \quad (5)$$

where  $V_i(\cdot)$  is the value function for grid  $i$ ,  $P_j$  is per bushel price of crop  $j$ ,  $W_P$  is per pound price of  $P_2O_5$ ,  $W_K$  is per pound price of  $K_2O$ ,  $\delta$  is a discount factor (i.e.,  $\delta = 1/(1+d)$  and  $d$  is the discount rate). The optimization is solved using a \$1.85 per bushel corn price, a \$4.30 per bushel soybean price, a \$0.24 per pound  $P_2O_5$  price, a \$0.13 per pound  $K_2O$  price, and a discount rate of 5 percent.

Return for a field under VRT application is equal to the sum of the discounted gross returns from all grids, which is given by:

$$V_{VRT} = \sum_{t=0}^{\infty} \delta^t \sum_{i=1}^n V_i(S_{i,P,t}, S_{i,K,t}) \quad (6)$$

Estimation of the returns is obtained using simulation [7]. The estimated gross margins are annualized using the following formula:

$$Annualized Value = \left[ \frac{Vd}{1 - (1+d)^{-n}} \right] \quad (7)$$

where  $V$  is the sum of discounted returns per acre,  $d$  is the discount rate,  $n$  is the number of years,

The process of calculating returns from VRT is repeated for each grid size, starting from the smallest size (0.625 acres) to the largest size (90 acres). Under uniform rate application, the whole field (90 acres) is treated identically with fertilizer applications based on the average soil nutrient levels for the entire field. This will be considered as Uniform Rate Technology (URT) with *full information*.

### 3. Results and Discussion

The steady states of soil P and K are shown in Figure 1 and 2, and demonstrate that P reaches the steady state level more rapidly than K. The estimated annualized gross margins from alternative fertility distributions are summarized in Table 1.

The estimated annualized gross margin is shown to increase with increased mean fertility levels. Increases in estimated annualized gross margins are more evident with higher mean soil P levels than for similarly high levels of K. One possible reason is that yield penalties for being below the steady state level are more severe for P than for K. In addition, more P is removed per unit of yield than is K.

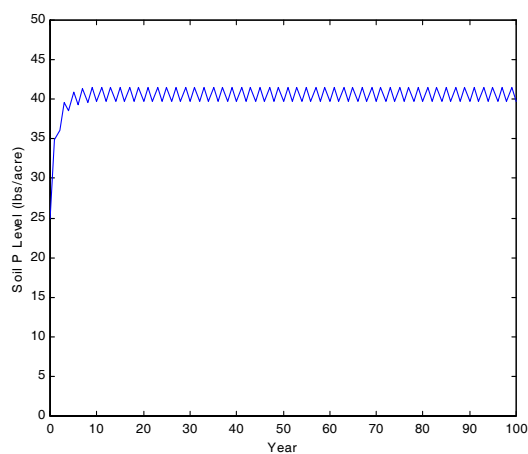


Figure 1: Soil P steady state levels

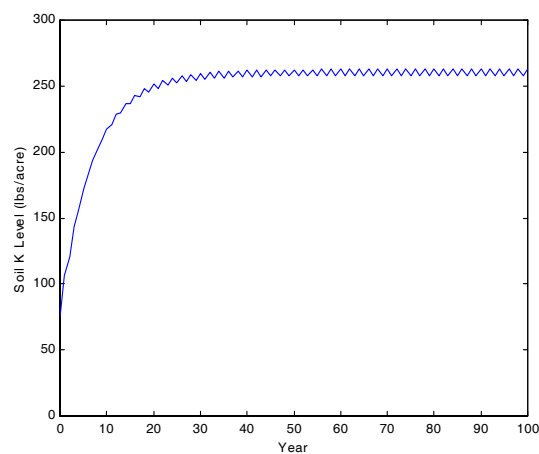
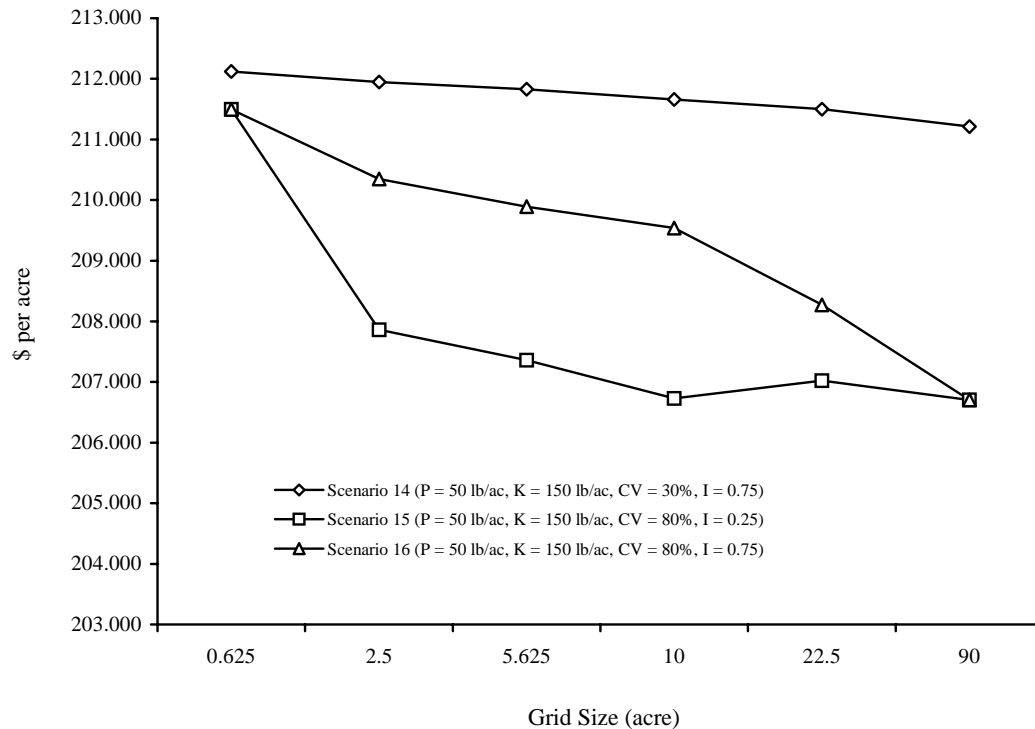


Figure 2: Soil K steady state levels

Table 1: Estimated Annual per Acre VRT Gross Returns from Alternative Fertility Distributions and Grid Sizes

Scenario	P		I' (P & K)	K		Grid Size					
	Mean (lb/ac)	CV (%)		Mean (lb/ac)	CV (%)	0.625 (acre)	2.50 (acre)	5.625 (acre)	10.00 (acre)	22.50 (acre)	90.00 (acre)
Annual Gross Returns (\$/Acre)											
1	25	30	0.25	75	30	208.192	207.974	207.898	207.853	207.832	207.805
2			0.75			208.192	208.126	208.075	208.031	207.939	207.805
3		80	0.25		80	207.560	206.224	206.151	205.816	205.799	205.685
4			0.75			207.560	207.192	206.779	206.500	206.369	205.685
5	25	30	0.25	150	30	209.317	209.004	208.904	208.808	208.789	208.770
6			0.75			209.317	209.232	209.148	209.058	208.814	208.770
7		80	0.25		80	208.724	206.815	206.336	206.396	206.100	206.050
8			0.75			208.724	208.038	207.759	207.259	206.707	206.050
9	50	30	0.25	75	30	210.990	210.464	210.416	210.283	210.276	210.239
10			0.75			210.990	210.885	210.730	210.690	210.497	210.239
11		80	0.25		80	210.329	206.983	206.177	206.010	206.214	206.307
12			0.75			210.329	209.728	209.073	208.859	207.583	206.307
13	50	30	0.25	150	30	212.120	211.582	211.378	211.292	211.261	211.211
14			0.75			212.120	211.945	211.829	211.660	211.500	211.211
15		80	0.25		80	211.496	207.860	207.359	206.729	207.023	206.706
16			0.75			211.496	210.350	209.891	209.538	208.273	206.706
17	75	30	0.25	225	30	215.062	214.031	213.777	213.599	213.557	213.538
18			0.75			215.062	214.774	214.614	214.341	214.088	213.538
19		80	0.25		80	214.387	209.888	208.083	207.212	207.292	206.899
20			0.75			214.387	213.362	212.011	210.832	209.864	206.899



**Figure 3: Estimated Annualized Gross Returns for Alternative Fertility Distributions and Grid Sizes**

For a given mean fertility level and with spatial autocorrelation constant, the estimated annualized gross margins are lower with higher soil fertility variation. This effect is associated with the nature of the yield plateau of the Mitscherlich-Baule production function. The value of yield gains for grids with soil nutrient values above the steady state levels are smaller than the increased cost of applied P and K in grids with soil nutrient values below the steady state levels.

For a given mean fertility level and with spatial autocorrelation constant, gross margins increase with decreased grid sizes, however, this effect is less pronounced with lower soil fertility variation. As the soil fertility is less homogeneous, treating each grid independently results in more gain, because grids with soil nutrient levels below and above the steady state levels can be identified.

As grid size increases, the aggregation effect becomes more prominent with decreased spatial autocorrelation. Even for the same degree of variability, gross margins decrease more rapidly with increased grid sizes as the

spatial autocorrelation decreases. In general, the benefit of breaking up the hypothetical field into smaller grid size is much more prominent with increased spatial variability associated with decreased spatial autocorrelation.

Figure 3 compares variation in gross margin for alternative grid sizes for three selected scenarios. Those scenarios have same mean fertility levels with different degrees of variability and spatial autocorrelation coefficients. It shows that given the same variability and mean fertility levels, aggregation effect is weaker for higher spatial autocorrelation (scenario 16) than that of lower spatial autocorrelation (scenario 15). Weak aggregation effect results in higher gross margins. However, given the same mean fertility values and spatial autocorrelation coefficient, gross margins are higher for lower CV (scenario 14) than that of higher CV (scenario 16). Low variation associated with high mean fertility levels may imply that the hypothetical field has more grids with soil nutrient values close to the steady state levels. As a result, reduced input costs can be expected.

Table 2: Estimated Annual per Acre VRT Net Returns with Grid Soil Sampling Every Third Year

Scenario	P		I' (P & K)	K		Grid Size					
	Mean (lb/ac)	CV (%)		Mean (lb/ac)	CV (%)	0.625 (acre)	2.50 (acre)	5.625 (acre)	10.00 (acre)	22.50 (acre)	90.00 (acre)
Annual Net Returns (\$/Acre)											
1	25	30	0.25	75	30	202.992	205.174	205.543	205.653	205.743	207.782 *
2			0.75			202.992	205.326	205.719	205.831	205.850	207.782 *
3			0.25			202.360	203.424	203.796	203.616	203.710	205.663 *
4			0.75			202.360	204.392	204.423	204.300	204.280	205.663 *
5	25	30	0.25	150	30	204.117	206.204	206.548	206.608	206.700	208.748 *
6			0.75			204.117	206.432	206.792	206.858	206.725	208.748 *
7			0.25			203.524	204.015	203.981	204.196	204.011	206.028 *
8			0.75			203.524	205.238	205.403	205.059	204.619	206.028 *
9	50	30	0.25	75	30	205.790	207.664	208.061	208.083	208.187	210.217 *
10			0.75			205.790	208.085	208.374	208.490	208.408	210.217 *
11			0.25			205.129	204.183	203.821	203.810	204.125	206.284 *
12			0.75			205.129	206.928 *	206.717	206.659	205.494	206.284
13	50	30	0.25	150	30	206.920	208.782	209.022	209.092	209.172	211.189 *
14			0.75			206.920	209.145	209.473	209.460	209.411	211.189 *
15			0.25			206.296	205.060	205.003	204.529	204.934	206.683 *
16			0.75			206.296	207.550 *	207.535	207.338	206.184	206.683
17	75	30	0.25	225	30	209.862	211.231	211.421	211.399	211.469	213.516 *
18			0.75			209.862	211.974	212.258	212.141	211.999	213.516 *
19			0.25			209.187 *	207.088	205.727	205.012	205.203	206.877
20			0.75			209.187	210.562 *	209.655	208.632	207.775	206.877

\* Highest Net Return

In general, variation and spatial autocorrelation in soil attributes are key factors in analyzing the optimum grid size. That is, if the field is not homogeneous and less spatially correlated, then applying VRT in smaller grid size has a potential value for significant profit relative to URT.

In spite of its benefit, there are some costs associated with VRT (i.e., sampling costs and application fee). By assuming cost of grid soil sampling of \$6.00 per sample and VRT application fee \$2.00 per acre surcharge over URT, table 2 shows the net economic gains of VRT. Soil samples are assumed to be taken every three years with sampling costs amortized over that period.

After adjusting for the application fee and grid sampling costs, VRT becomes less promising, particularly for scenarios with low variability and low mean fertility levels. Increase in crop yield from treating each management zone independently might not cover the extra costs associated with VRT.

In scenarios with low variation of fertility levels (CV = 30%), URT was the optimal strategy in all scenarios. Generally, VRT was identified as the optimal strategy only in scenarios that displayed both high variability and high mean fertility level. In the case where variability and mean fertility level are significantly high associated with low spatial autocorrelation, VRT

produces greater net return than URT, even with the smallest grid size to base the input application decisions, as in scenario 19. As the soil attributes are less homogeneous in value and less spatially correlated in space, treating the field in smaller grid size results in more gain. On the other hand, as the spatial autocorrelation increases, soil attributes that are close in space become more similar; in this case, larger grid size result in higher net return, as in scenario 20.

The analyses in Table 2 highlight the importance of data collection and application fees in the economics of VRT. Clearly, VRT is only profitable under selected conditions, and the optimal grid size will vary with these conditions. It is worth noting that, if sampling costs per acre per year were reduced, or if these costs were spread over other variable rate input decisions each of which could produce a small improvement in gross crop value, then the optimal decision would more frequently include VRT and the optimal grid size would be smaller. For instance, if the sampling/analysis costs were cut in half (from \$6 to \$3 per sample) from those used in Table 2, VRT would become optimal 50 percent more often; scenarios 11, 12, 15, 16, 19 and 20 would be best with VRT under this reduced cost assumption. These results underscore the potential positive impact on profitability of VRT of any new technology that will reduce the costs of soil sampling and analysis or VRT application services.

#### 4. Conclusions

It is well understood that variation in soil attributes is a necessary condition for the profitable application of VRT. That is, if a field is homogenous, then the optimal input application will not vary across the field and VRT creates no value. The results of this study demonstrate that not only is variation important, but there is an important interaction among variation, spatial autocorrelation, and mean level of soil attributes such as fertility.

In general, the benefit of breaking up the field into smaller grid size is much more prominent with increased variability associated with decreased spatial autocorrelation. This is the case where VRT has a potential for significant profit relative to URT. However, the application fee and grid soil sampling costs associated with it may cause VRT only profitable under selected conditions.

In the case where variability and mean fertility levels are significantly high associated with low spatial autocorrelation, VRT produces greater net return than

URT, even with the smallest grid size to base the input application decisions. As the soil attributes are less homogeneous in value and less spatially correlated in space, treating the field in smaller grid size results in more gain. On the other hand, as the spatial autocorrelation increases, soil attributes that are close in space become more similar; in this case, VRT with larger grid size result in higher net return than URT.

Intensive grid soil sampling is the primary method used today to determine VRT fertilization rates. However, intensive soil sampling is relatively costly. It may be possible for producers to utilize a GPS-equipped yield monitor to generate yield maps. Although they will not get information about soil nutrients from yield monitoring, yield patterns may be a useful indicator of spatial autocorrelation of key field characteristics. This may allow the producer to make an informed judgment about whether an investment in intensive soil sampling might be warranted, and what grid size might be appropriate. This suggests that a fruitful area of further research may be techniques to identify appropriate management zones using yield data.

Study results also suggest that new technologies or practices that lower soil sampling/analysis costs will substantially increase the likelihood that VRT will be profitable. A halving of the soil sample costs resulted in an increase of more than one-third in the number of scenarios where VRT was profitable. It also resulted in smaller optimal grid sizes in a number of scenarios.

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