REPRESENTATION OF SM OPERATORS ON SEQUENCE SPACES WITH A NEW NORM

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Abstract. We give some representations of SM-operator on sequence spaces ,1 < < ∞using a new norm different from that of ones in the beginning and some examples are given.

Keywords: Norm, SM-operator, Sequence Spaces

1. INTRODUCTION

This paper is a continuation and modification norm of SM-operator on Banach Spaces [2] and [3] by modifying one of the most important class of bounded operators called Hilbert-Schmidt operator. This also have been done in different way in [1].The papers are also motivated by some application in statistics [6] and in Physics [4]. The work is positive whenever we preserve some instrinsic properties of Hilbert-Schmidt operators i.e, reflexivity and separability of the Banach spaces. The point is, reflexivity and separability of the Banach spaces are to guarantee the existence of its bases with which used to construct the new operator.

2. PRELIMINARY

Let , be reflexive and separable Banach spaces and , be their duals. For any and , let ( ) be denoted by (,) and vice versa. It is observed that a linear independent sequence { } is called a bases of if for each vector , there is a unique scalars { } such that = ∑ . For, ‖ ‖ = 1 for each , a sequence { } is said to be biorthonormal with respect to a bases ( ) if (,) = , where = 1 for = and = 0 for ≠ . If the pair { }, { } is biorthonormal system of then (,) = for each .

Let (,) be a collection of all linearly continuous operator from into . If (,)then (,), where is adjoint of . Hence, if { }, { } are two bases of and { } is a bases of , then every (,), we have

( ) = , ( )

= (( ),)

and

( ) = , ( )

= (( ),)

Hence, if { }, { } are two bases of and { } is a bases of and (,), then we have

(( ),)

= (( ),)

Definition 2.1 [3] *An operator* (,) *is called SM-operator from a Banach space into , if*

(( ),) < ∞

for every bases { } *and* { } .

This condition gave a norm in the space of collection of all SM-operators from Banach space into by

‖ ‖ = ∑ ‖∑ (( ),) ‖.

The norm is more stronger than that of one in Darmawijaya et al, 2006 and Ansori et al,2008by using

‖ ‖ = |(( ),)|

3. MAIN RESULT

Let (,) is a collection of all SM-operators from Banach space into .

Theorem 3.1 *For every (,) we have*

(i). ‖ ‖ ≤ ‖ ‖

(ii). (,) *is a normed Banach Space.*

Proof:

(i). Let { } be a bases of and { } be a bases of . For every , we have

‖ ( )‖ = (( ),)

≤ ‖ ‖ ‖ ( )‖

= ‖ ‖ (( ),)

= ‖ ‖‖ ‖

or‖ ‖ ≤ ‖ ‖ .

(ii). (,) is a norm space with respect to ‖ . ‖ , because:

(ii.a) For every (,), we have

‖ ‖ = (( ),)

≥ 0

and

‖ ‖ = 0

(( ),)

(( ),) = =

(isnull vector) for every m

= (O is null operator)

= (O is null operator)

(ii.b) For every (,) and scalar , we have

‖ ‖

= (( ),)

= | | (( ),)

= | |‖ ‖

(ii.c) If , (,) , then

‖ + ‖ = (

+)( ),)

= (( )( )

+ ( )( ),)

= (( )( ),)

+ (( )( ),)

≤ (( )( ),)

+ (( )( ),)

or

‖ + ‖ ≤ ‖ ‖ + ‖ ‖

Based on ii.a, ii.b, danii.c the space (,) is a normed space with respect to norm ‖ ‖ . Now, for the completeness of (,), let { } is any Cauchy sequence in (,). For every real number > 0, there is a positive integer such that for every two positive integer , ≥ we have

− < −.

We will prove that there is (,) such that

lim ‖ − ‖ = 0.

Since,

− ≤ − < $\frac{}{2}$

for every , (,) with , ≥ , then the sequence { } is also a Cauchy sequence in (,) such that lim = . Hence,

((−)( ),)

= lim (

−)( ),)

= lim − ≤ $\frac{}{2}$

for every positive integer $\geq$. It has shown that $-$ (,), for $\geq$ and $=$ $-$ $+$ (,). Hence, { } is convergent to or lim ‖ $-$ ‖ = 0and (,) is a Banach space.

Now let = and = , $1 <$, $< \infty$, $- + - = 1$. Sequence with $1 <$ $< \infty$ are spaces of vectors given by

$$= \bar{} = \{ \}: \quad | \ | < \infty$$

and = , $<$, $< \infty$, $- + - = 1$ with norm

$$\| \| = \quad | \ |$$

and

$$(\bar{},) =$$

for every $\bar{}$, . Since have bases and to simplify we can take standard bases of i e., { $\bar{}$ } where

$$\bar{} = 0,0,0,\ldots,0,0,0, \ 1 \ ,0,0,0,\ldots \quad ■$$

Theorem 3.2[5] *Let and* , $1 <$, $<$ ∞, $- + - = 1$. *A linearly continuous functional of infinite matrix* = *from into if and only if*

$$< \infty$$

Theorem 3.3 *A linearly continuous operator* : , $1 <$, $< \infty$, $- + - = 1$ *is an SM-operator if and only if there is a infinite matrix satisfying:*

i. $\bar{} = \sum$ *for every*

ii. $\sum \ \sum \quad < \infty$

iii. $\sum \ \sum \quad < \infty$

Proof:

( )Since : , $1 <$, $< \infty$, $- + - = 1$ is linearly continuous operator then by Theorem 1.2 automatically (i) and (ii) hold. Since is an SM-operator then we have

$$= \qquad < \infty$$

( ) Based on (i) and (ii) we have : , $1 <$, $< \infty$, $- + - = 1$ is linearly continuous operator and based on (iii) we have

$$\| \| = \qquad = \qquad < \infty$$

.

The proof is complete.

For examples: Let matrix , , = 1,2, ... with

$$= \frac{1}{} \qquad = \qquad 0 \quad \text{others}$$

= : is an SM-operator since:

(i) For every $\bar{}$ we have

$$\| \| = \qquad = \qquad < \qquad < \infty$$

So we have $\bar{}$ for every .

(ii) $\sum \ \sum \quad = \sum \quad < \infty$

(iii) $\sum \ \sum \quad = \sum \quad < \infty$

■

4. CONCLUSION

A linearly continuous operator : , $1 <$, $< \infty$, $- + - = 1$ is an SM-operator if and only if there is a infinite matrix satisfying: (i).

$\bar{ } = \sum$ for every $\bar{ }$

(ii). $\sum \sum < \infty$ and (iii). $\sum \sum < \infty$.

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