OBJECT-ORIENTED APPROACH FOR THE ASSESSMENT OF MOMENT-CURVATURE RELATIONSHIP OF A VARYING-WIDHT AND MULTI-MATERIAL BEAM CROSS SECTION

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ABSTRACT

This research developed the rational approach to shear design in 1984 Canadian Code Provision into a new approach which is object-oriented in fashion, and presented it for the purpose of assessing moment-curvature-relationship of a varying-width and multi-material beam cross section. Unlike the traditional method that views a cross section as a single entity, this new approach views a section as a composition of autonomous objects. In this approach, a cross section is recognized as a system which is made up of objects, of which each can be predicated uniquely; behave autonomously in responding to loading, and capable of communication between each other. Being in such a fashion, the approach was shown to be capable to faithfully represent a section which varies in width, and is made up of materials with different mechanical characteristics, in whatever possible arrangement. To compensate for the painstaking computation that may be involved in the approach, and maintain its object-oriented fashion, an-object-oriented computer-software that uses an object-oriented user interface platform was recommended to be provided as an auxiliary to the approach.

Key Words: composite section; computer software; layers; moment-curvature-relationship; multi-material section; new approach; object-oriented approach; object-oriented computer-software; varying-width cross section.

Overall historic capacity of a beam section loaded in flexure, from incipient loading until full exhaustion of its capacity is best described by Moment-Curvature-Relationship. This relationship is generally presented in a diagram called Moment-Curvature-Diagram (MCD), which shows variation of flexural capacity of a section, as curvature of the section increases from minimum to ultimate. Being as such, an MCD serves as an important tool in the hand of a structure designer to look into the variation in flexural-capacity of a section, when designing for the strength of a beam component of a structure. In practice, a structure-designer will first propose a section with certain characteristics and then assesses the moment-curvature-relationship of the section to gain knowledge about the overall flexural capacity of the same. In case of deficient in capacity, the designer will modify the characteristic of the section as to meet the required capacity, passes the modified-section under an analysis to assess its moment-curvature-relationship, then again investigate the capacity of the section as before. This is repeated several times until the designer arrives at a section whose flexural capacity complies with the requirement. From this it can be

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inferred that moment-curvature-relationship of a section stands in critical node as an important platform in the process of strength-design of a beam component of a structure.

Currently, standard method for assessing moment-curvature-relationship of a beam-section loaded in flexure is the one which is generally known as strength-section-analysis, at which zero strain and strain of certain magnitude are respectively introduced to neutral fiber and one of the extreme fiber of a section. Curvature of the section and then flexural stresses at extremes fibers can then be calculated, and based on those stresses, internal forces are computed. This then leads to the computation of the section's flexural-moment capacity. This method is traditional. It has been introduced and used in practice since the beginning of modern period of structure-strength-analysis (Kinney 1957) and is the prevailing method that appears in standard text books, taught to civil-engineering students. Some of the prominent authors who presented the method are E.P. Popov and R Park and T Paulay. Popov used moment-curvature relationship in analyzing cyclic loading of steel beams and connection (Popov and Bertero 2011), whereas Park and Paulay used it in their book when discussing ductility of reinforced-concrete beam, and presented it as a standard method for assessing moment-curvature relationship of a reinforced-concrete beam-section (Park and Paulay 1974).

Despite its wide use, this method has limitation that can readily be felt when one applies it on section of a more complex shape. As shape of a section departs from being a simple rectangular towards a more complex I, T, C, L, Z, circular and tubular/box, variation of width along the depth of the section demands that for every stage of curvature, internal force should be separately calculated for each part of the section that has a different width. This makes the calculation cumbersome and will be worse at application of the method on section with continual varying width such as circular or trapezoidal section. At these situations, practicing engineers who design for the strength of structure tend to do 'approximation' by neglecting variations of width, hence result in either very conservative or defective design. Limitation of this method will become more pronounced, when, in addition to varying width, the section in question is made of materials with different tensile and compressive characteristic -such as concrete-, and when, as often the case, shear and torsion are taken into account in addition to flexural-moment. Furthermore, with the recent emergence of composite and 'sandwiched' beam, where a section is made up of several materials, each with different mechanical properties, this method is proved to be disadvantage. All these, undoubtedly call for a new method for strength-section-analysis that takes into account in a more realistic way, variation of width and of characteristic of materials that make up a section.

A more realistic way to take account of variation of width and of material compositeness of a section can be achieved by approaching the section, not as a single entity as in the traditional method, but as a composition of a group of objects, of which each of the object is interrelated in specific manner one to the other. This way of approaching is new and herein called 'an object-oriented-approach'. In this approach, a section is recognized as a system which is made up of objects. Each object can be predicated uniquely, behave autonomously in responding to loading, and is capable to communicate results of its behavior to its neighboring objects. One may already notice that this approach is a radical departure from the traditional one, and that by regarding a section as a system of autonomous-communicating objects as just presented, minute variation of width and of mechanical characteristics of a section is adequately and realistically taken into account in the analysis. It can be mentioned here that on the approaching of a section in object-oriented manner, lays the strength of this new approach over the traditional one.

A close match to this new approach is 'the rational method' presented for the 1984 Canadian Code Provision by Collins and Mitchell (1980), for flexure and shear analysis of a reinforced-concrete and prestressed-concrete beam section. In it, prior to calculation, a section is first divided into a definite numbers of horizontal layers of identical thickness. While identical in thickness, each layer varies in width, in accordance with variation of width of the section. Unto each layer is assigned mechanical-characteristics, mostly in stress-strain format, in accordance with mechanical-properties of materials represented by each layer. In its application, to each layer, a strain whose magnitude is in consistence to the distance of the layer to one of the extreme fiber of the section is introduced. Calculation of curvature of the section is then done based on each of these strains, and then, based on particular stress-strain format of each layer, stress at each layer, is calculated. These stresses collectively serve as the basis on which the last step namely computation of flexural-moment capacity of the section is done; thereby flexural capacity of a section, at certain stage of curvature, is obtained. This is repeated several times, each time with an increment of curvature, until full moment-curvature relationship of a section is obtained. It can be seen here, that more accurate results can be achieved by having more numbers of layers for a section; and as the number of layers approaches infinity (hence, thickness of each layer approaches zero), detail variation of width and of characteristic of materials forming the section is more faithfully represented and taken into account. A highly accurate calculation is thus achieved. Rationality of this method and its capability of application on section made up of materials with different mechanical characteristics were already felt by the later cited authors themselves, who presented it in their later publication (Collins and Mitchell 1986) as a rational method for strength analysis that includes prestressed-concrete, in addition to reinforced-concrete. Seeing the same capability, Tjandra and Pah (1993), and Pah (1999) developed a procedure based on that method for flexure and shear design of reinforced concrete, and expanded its application to include high-strength-concrete. Rational of the method for assessing moment-curvature relationship was seen specifically by Wennyi (2006), who designed a computer-software based on the same method, for calculation and graphical presentation of moment-curvature relationship of a reinforced-concrete beam-section. Later mentioned work has contained specimen of object-oriented approach, as the author made room in the software he designed for predicating each layer differently in regard to stress-strain relationship.

However, in all works cited in the preceding, none of the authors specifically view the layers as autonomous-communicating objects. They merely regard the layers as smaller fraction of a section. Moreover, communicating capability of each layer was obviously not recognized. A section is nevertheless seen as a single entity, and not as a system of interrelated objects. These limit the method so developed from application on a multi-material section, where section of a beam is made up of materials with different mechanical characteristics, arranged in a rather complex manner; and each is capable of behaving in completely different way. In view of this, this research developed anew the rational method presented originally by Collins and Mitchell (1980) into 'an object-oriented approach', and presented it as a tool for the purpose of assessing moment-curvature relationship of a multi-material beam cross section, with varying width.

CONSTITUTION OF THE APPROACH

Since the approach is an object-oriented in character, it is worthwhile to first presenting classes of objects that constitute this new approach, and only secondly presenting the general way by which this approach operates, after all classes of object have been thoroughly presented.

CLASSES OF OBJECTS

In this approach, an object is a conceptual component of a section that has a clearly defined function and plays a certain role. Moment-curvature relationship of a beam's section will be generated and presented when a definite system of objects works together for the beam's section in question. Objects of similar kind form a class of objects.

Curvature-Generator

First class of objects in this approach is the Curvature-Generator. A curvature-generator is an object that generates curvature in a successive increasing magnitude, from zero to a user-defined maximum, positive or negative in direction, at a user-defined increment.

Layers

Second class of objects is a layer. A layer is a conceptual subdivision of a beam's cross-section into smaller parts. Figure 1 shows two sections, a rectangular and a circular section respectively, which are subdivided into a number of layers. As might be inferred from the figures, arranging the layers in a certain manner and interrelate them in a certain way, form the original section from which those layers are subdivisions. In this approach, each layer is regarded as an object by itself that behaves autonomously. A layer has several parts, each with a clearly defined function.

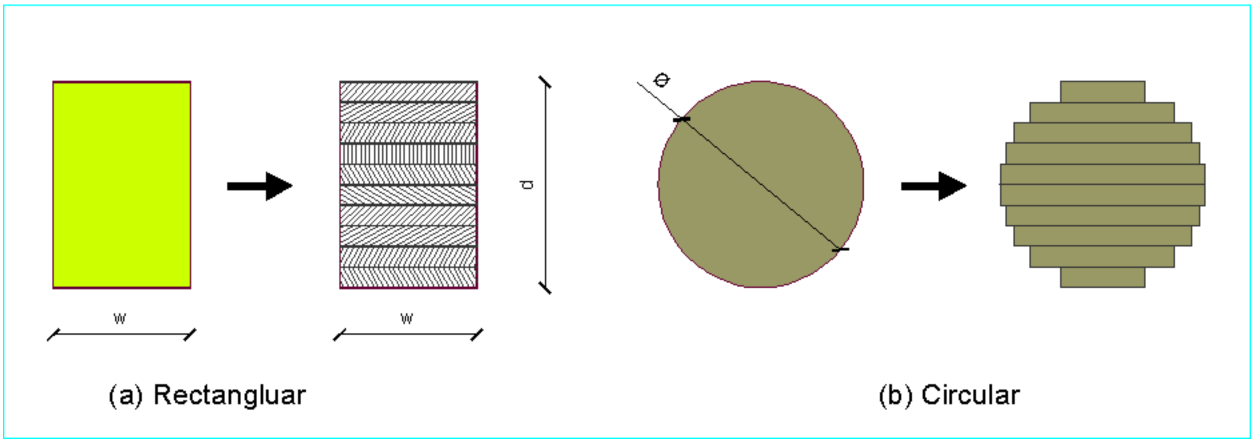

Figure 1. Subdivision of a Rectangular and a Circular Cross Section into Object of Layers

Figure 2 shows parts of a layer. As shown in the figure, first part of a layer is Identifier. An identifier identifies a layer. Since layers of a section are numbered from top to bottom, a top layer is always indicated as L=1 by the Identifier, and an i^{th} layer from the top is indicated as L=i by the same. User shall input Numbers of Layers [NOL], and depth (d) and width function (w) of the considered section into the system. In case of section has width that varies along its depth, a mathematical equation of width as a function of depth ($w_i = f(d)$) should be assigned to the identifier instead of a constant function. Upon the input of those information, the identifier generates the width w_i of a layer, in accordance with the function, thickness t_i of a layer as $t_i = \frac{d}{[NOL]}$, and position of a layer in respect to top fiber j_i as $j_i = j_{i-1} + 0.5t_i$.

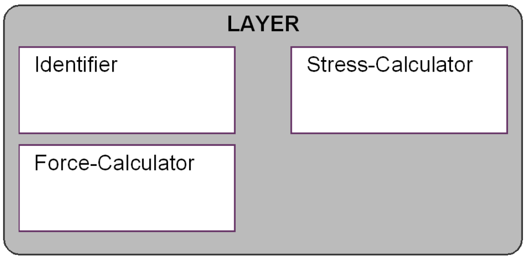

Figure 2. Parts of a Layer

Second part of a layer is Stress-Calculator. Unto a stress-calculator is assigned a mathematical function that relates a strain to a flexural-stress, in particular way, consistent with mechanical characteristic of material represented by the layer. In general, the function will take the following form:

$$S_i = f(\varepsilon_i) \tag{1}$$

where:

S_i is flexural stress at i^{th} layer, and

ε_i is strain in i^{th} layer, negative and positive for compressive and tensile respectively.

Third part of a layer is Force-Calculator. A force-calculator calculates force in a layer that is due to the stress in the same layer. The force is calculated in the following way:

$$F_i = S_i w_i t_i \tag{2}$$

where

F_i is the force in the i^{th} layer, negative for compression and positive for tension;

S_i is the stress in the i^{th} layer, negative for compressive and postive for tensile;

w_i is width of the i^{th} layer, and

t_i is thickness of the i^{th} layer.

Within this class of object, three kinds of layer can be distinguished namely Top Layer, Interior Layer and Bottom Layer

Top Layer

A top layer is the layer that contains top fiber of a section. It is similar in kind with a layer as defined in the preceding, except only for two facts. First, in addition to common parts of a layer, a top layer has Strain-Generator. A strain-generator generates strain in succession increasing magnitude at a user-defined increment. Secondly, identifier of a top layer always assign $j = 1$ as the position of the layer. Figure 3(a) shows parts a top layer.

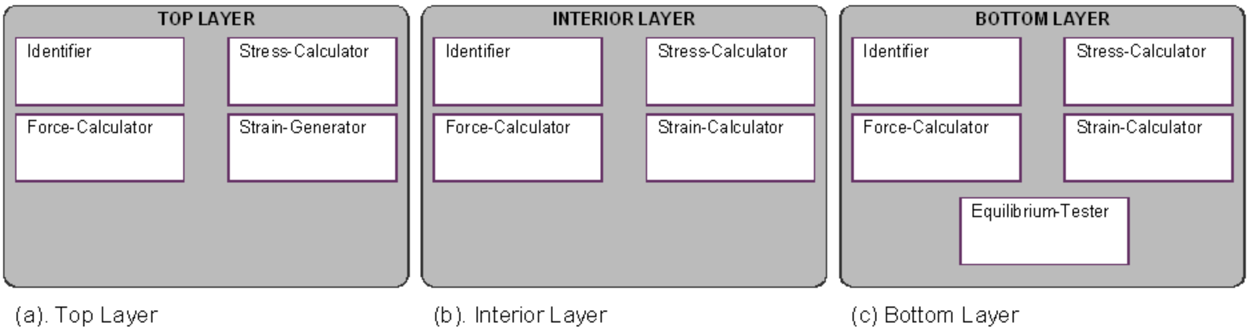

Figure 3. Parts of a Top Layer, an Interior Layer and a Bottom Layer

Interior Layer
An interior layer is a layer that contains interior fibers of a section. An interior layer is similar in kind with a common layer, except for the fact that in addition to common parts of a layer, an interior layer has Strain-Calculator. A Strain-Calculator calculates the strain in the layer in the following way;

$$\varepsilon_{L=i} = \varepsilon_{L=i-1} - \phi_1 t_i \quad (3)$$

where

$\varepsilon_{L=i}$ is strain of the i^{th} layer;

$\varepsilon_{L=i-1}$ is strain communicated to a layer from layer immediately at the top the layer in question;

ϕ is trial curvature generated by the Curvature-Generator, and

t_i is thickness of the i^{th} layer,

and registers it as the strain of the layer. Figure 3(b) shows parts that make up an interior layer.

Bottom Layer
A bottom layer is the layer that contains bottom fiber of a section. A bottom layer is the same in kind with an interior layer as defined in the preceding, except for the fact that in addition to common parts of an interior layer, a bottom layer has Equilibrium-Tester. An equilibrium-tester checks and indicates whether or not a section's internal forces are in equilibrium. Figure 3(c) shows parts that make up a bottom layer.

Moment-Calculator
Moment-calculator is a class of object that calculates the internal flexural-moment of a section, for every stage of the process.

Moment-Curvature Platform
The last class of object in this approach is the Moment-Curvature-Platform. Moment-curvature-platform registers flexural-moment and curvature of a section at each stage of the process, generates an MCD in relevancy with the registered flexural-moments and curvatures, and publishes it.

OPERATION OF OBJECTS

Having discussed classes of objects in the preceding, we come now to the presentation of how the approach operates. A steel-wood composite beam cross section of trapezoidal shape shown in Figure 4(a) will be set as an example upon which the approach is applied, and will be referred to throughout the presentation for the purpose of giving numerical and graphical illustration. Arrangement of material composition of the given section is as shown in the figure.

Preparatory phase of the operation starts with user eliciting depth, width function, and stress-strain format for each of the material that composes the given section; and proposing a number of layers [NOL] to the system. The NOL should be such as to be sufficient to faithfully represent the variety of width and of material composition of the section. It is clear from Figure 4(a) that depth of the section (d) is 400 mm, top width is 100 mm, and bottom width is 300 mm; hence the width function is:

$$w_{(d)} = 100 + 0.5d \quad (4)$$

of which

$w_{(d)}$ is width of the section, in mm, as a function of its depth, and

d is depth of the section, in mm.

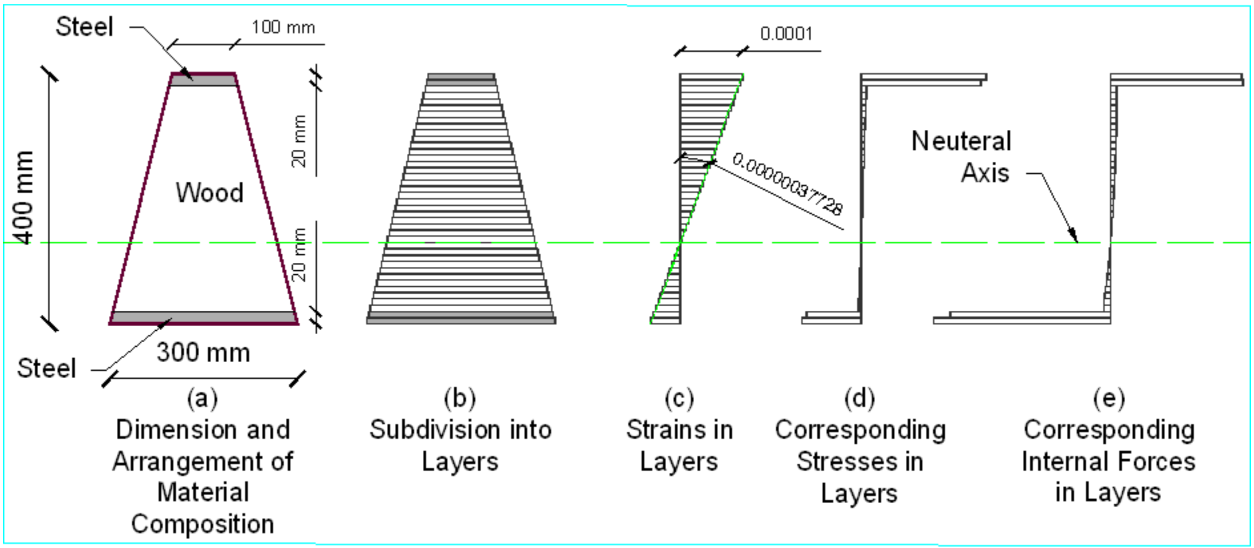

Figure 4. Dimension, Material Arrangement and Subdivision into Layers; and Status of Internal Forces of the Sample Section, at MCD Coordinate of [M = 1.98 kNm; φ = 3.7728 x 10^{-7}]

Since the cross section is composed of two kinds of material, two stress-strain formats are required, one for steel and the other for wood respectively. Let us have for steel, the following stress-strain format:

$$S = 200000\varepsilon \quad \text{for } -0.0012 \le \varepsilon \le 0.0012 \quad (5a)$$

$$S = 240 \quad \text{for } -0.048 \le \varepsilon < -0.0012 \text{ and } 0.0012 < \varepsilon \le 0.048 \quad (5b)$$

which is an idealized bi-linear stress-strain format of f_y = 240 MPa steel (Setiawan 2008), and for wood, the following:

$$S = 9800\varepsilon \quad \text{for } -0.006 \le \varepsilon \le 0.006 \quad (6)$$

which is a stress-strain format of Yellow-Meranti Timber (Ngu 2007). Later in the process, these two stress-strain formats will be assigned to each of the layer, in accordance with the material each of them represents. Following the eliciting of section's characteristics, shall be the input of the depth of the section d, and width function $w_{(d)}$, and the proposing of NOL into the system. At the input of these information, the system will generate layers of identical thickness and particular width (in accordance with the width-function), in numbers according to the NOL, and arrange them in successive order from top to bottom. Figure 4(b) shows the arrangement of layers. Note that since NOL = 40 has been proposed, the section was divided into 40 layers, 36 of which are interior wood layers, whereas the top and 2^{nd}, as well as the 39^{th} and the bottom are steel layers. Unto the top, 2^{nd}, 39^{th} and bottom layer therefore were assigned stress-strain format of steel (Eq. (5)), while to the rest were assigned that of wood (Eq. (6)). These conclude the preparatory phase of the operation and pass it to the analysis phase.

At the initial stage of the first round of the analysis phase, both strain in the strain-generator of the top layer and curvature in the curvature-generator are set to zero. Analysis process then starts with strain-generator in the top layer generates an initial strain and the curvature-generator generates a trial-curvature. Top layer will register the strain so generated as its strain, and based on that strain, calculates stress in the layer in consistence with the assigned stress-strain format, and then calculates internal force in the layer by way of Eq. (2). In this way, internal force in the top layer, which is based on the initial strain, and in consistence with the mechanical characteristic of material represented by the layer, is obtained. Following it, the top layer communicates its strain to the layer immediately below it (the second layer). At the communication of the strain from the top layer, the second layer will generate its strain by way of Eq. (3), registers it as its strain and calculates for the stress and internal force in the layer, in the same way as that done by the top layer, described in the preceding. In this way, the internal force in the second layer is obtained. Second layer will communicate its strain to succeeding layer (the third layer), which will calculate stress and internal force in the same way as described above, and so forth until internal forces in every layer have been obtained. Figure 4(e) shows internal forces in layers of the given section, at top layer strain of 0.0001 mm/mm and initial curvature of 3.7728×10^{-7} /mm. After internal forces in all layers have been obtained, the equilibrium-tester in the bottom layer will check whether or not those internal forces are in equilibrium. At non-equilibrium, the curvature-generator will increase, or decrease the trial curvature by an increment, and send it to interior and bottom layers, as the basis on which layer-strain and internal-forces are recalculated from layer to layer in the way as described previously. Curvature-generator will do this, increasing or decreasing in a definite increment, until it arrives at curvature that produces equilibrium of internal forces in the section. At equilibrium, the moment-curvature-platform will register the curvature as the abscissa, and the corresponding internal flexural-moment calculated by moment-calculator, as the ordinate of a point in the MCD of the section in question. Figure 4(e) shows internal forces in layers which were results of the analysis for the registered MCD coordinate of M = 1.98 kNm and Curvature φ = 3.7728 x 10^{-7} /mm. Registration of a coordinate for the MCD concludes a round of analysis. The system then passes into the next round.

Next round of analysis starts by having the strain-generator in the top layer increases the initial strain by a positive increment, and run the whole process in the same way as described above, until it arrives at the next coordinate of MCD. The process runs continually in the same manner as described above; every time at the end of a round, the moment-curvature-platform registers moment and curvature, until ultimate curvature of the section has been reached, and thereby an MCD of the section is obtained. As the final step, the moment-curvature-platform publishes the obtained MCD for the section in question.

DISCUSSION

Since the constitution of the approach has been presented in the preceding, we come now to discussion about the strength this approach may offer, in compare with that of the traditional one. First, advantage of this new approach will be highlighted, and then secondly, the possible development of this approach in days to come to enhance its strength is presented.

ADVANTAGES OF THE NEW APPROACH

There are five points this new approach offers in advantage to that of the traditional methods of section-analysis.

Representing Variation of Width of a Cross Section more Realistically

First, it can almost instantly be seen from studying this new approach that by utilizing object of horizontal layers which varies in width, minute variation of width of a section is more accurately and realistically represented. This approach is therefore far more appropriate than the traditional method, for the application on sections with changing width, such as I, T, C, L, Z, circular and tubular/box shape. A more realistic representation of section width can still be achieved by utilizing as many as possible numbers of layer (NOL). As the NOL approaches infinity, real and minute variation of section-width is more truly taken into account.

Representing a Composite Section more Faithfully

Secondly, by utilizing layers, compositeness of a section can be more realistically represented and taken into account. Since each layer can be predicated to behave mechanically as a different material, utilizing as many as possible [NOL] makes the approach capable to faithfully represent a cross section which is composed of as many as possible different materials that are arranged in whatever configurations. Capability to faithfully representing a high-degree composite cross section of a beam, as herein highlighted, is an advantage of this approach, which is obviously not shared by the traditional one.

Representing Materials of Different Tensile and Compressive Mechanical Characteristics

It follows from discussion in the preceding that a layer can be predicated to behave differently and in particular way, when it is under tensile strains from when it is under compressive strains. This allows the approach to represent, in a more realistic way, a composite section whose parts are made of concrete or other materials with different tensile and compressive mechanical properties -reinforced concrete is an example-. This again is a distinctive capability of this object-oriented approach, where the traditional one is found to be wanting.

Adaptability to Employ Results of Future Study and Research

Moreover, ability to predicate a layer to behave as a material in a particular way makes the approach flexibly adaptable to the future knowledge about mechanical characteristics and strength of materials. Should future studies and researches reveal a new knowledge about mechanical characteristic and strength of materials, or should it invent a new material with its particular mechanical characteristic, user of this approach can easily employ the new knowledge or material into his/her analysis by simply employing them in predicating the layers. Flexible adaptability to new knowledge is an advantage, even a strength of this approach, which is far over the traditional methods of section-analysis, especially over those which are empirical in nature, such as those for shear, which limit its application on condition of experiments from which they were construed.

More Realistic Insight about the Way a Section Behaves in Response to Loading

Finally, since being heavily formatted as a procedure, the traditional methods of section analysis, fail to give to users insight as to how parts of a section behave in response to a scheme of loading. At their best, traditional methods, especially those which are empirical in nature can only give to user a certain routine to analyze and design a section. Unlike the traditional ones, this approach, by approaching a section in object-oriented way, is capable to give insight to the users on how conceptual parts of a cross section behave as a system in giving response to a scheme of loading, and thereby provides way to users to understand the section more adequately.

This in turn paves way to the user to manipulate characteristics of the section more realistically, and to come up with a more appropriate design.

POSSIBLE DEVELOPMENT OF THE APPROACH

Apart from the advantages enjoyed by this approach, there are points in it that need solutions and developments to further enhance its power. Followings are those points.

Object-Oriented Computer Software

One may immediately see that there is, contingent with having objects and layers in a big number, a rigorous computation that will be proved disadvantage if it should be done manually. In view of it, this research recommends that a computer-software be developed to aid in the whole process of computation. In order to maintain its object-oriented characteristic, that computer software should be developed in an object-oriented philosophy and object-oriented programming language; and to faithfully giving realistic insight to users as to how parts of the section behave, the computer software should as well use an object-oriented interface platform.

Shear and Torsion Analysis

Furthermore, one may also note that the object-oriented approach currently presented by this paper had not made room for the analysis of shear and torsion. Since at most cases, shear and torsion concurrently present in a section together with flexure, design of layers in this approach should be developed further as to enable predicating layers for behaving in response to shear and torsion strain, in addition to that of flexure.

Slippage at Interfaces of Different Materials

Moreover, in reality, at every composite section, there is slippage between interfaces of different materials. Currently, the approach does not take this into account. To take this into account, this approach should be further developed, either by inventing a new class of object, or by characterizing layers as to capable of taking into account of slippage between interfaces of different materials. It can be asserted in this connection that consistent development of the approach for slippage between interfaces of different material can carry the approach further as a tool for analysis and design of shear connectors of a composite beam.

Biaxial Analysis

This approach currently utilizes principle object in the form of a layer of identical thickness that extends horizontally. Since it extends horizontally, the approach can only take account of the variation of

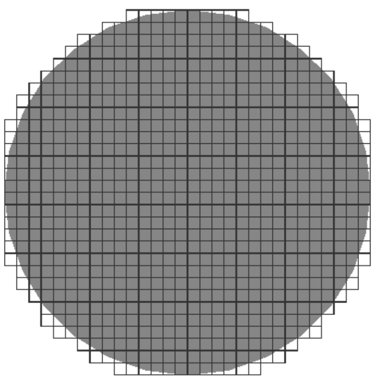

Figure 5. Subdivision of a Circular Cross Section into Objects of Square

width and of composition of materials along the depth of a section, and can only perform analysis for strength of a section uni-axially, that is only on its major axes. To enhance the approach as to capable of taking into account variation of depth and of material composition along the width of a section; and to analyze for strength of a section on both of its orthogonal axis (biaxial analysis), a subdivision in the form of a square should be utilized instead of in the form of a horizontal layer. Figure 5 shows an example of subdivision of a circular section into objects of square. It can be seen from the figure that by having square objects in a big number, variation of depth and of material composition along both of the width and depth of a section can be faithfully taken into account, and biaxial analysis can be performed.

CONCLUSION

1. In place of the traditional method of section strength-analysis that approaches a section as a single entity, this research presented a new approach which is object-oriented in nature that approaches a section as a composition of a group of autonomous objects.
2. This new approach recognizes a section as a system which is made up of objects, of which each can be predicated uniquely, behave autonomously in responding to loading, and communicate to each other.
3. Principle object of the approach are layers which are conceptual subdivision of a section into smaller parts, along its depth. Each layer has identical thickness, but varying in width, in accordance with variety of width of a section. Unto each layer is assigned a particular stress-strain format, in consistence with material represented by the layer.
4. Over against the traditional method, this new approach has several notable advantages. First, by recognizing a section in object-oriented fashion, variation of width, of characteristics of material that make up a section, and of arrangement of those materials within a section, are more faithfully taken into account in the analysis; secondly, the approach can take into account materials with different tensile and compressive mechanical characteristics such as concrete; thirdly, it is flexibly adaptable to results of future study and research in regard to strength of materials; and finally it gives insight to users as to the way various parts of a section behave in response to a scheme of loading.
5. To enhance the strength of the approach, this research recommends that an-object-oriented computer-software be prepared to aid in the computations; that the approach is further developed, to include both shear and torsion analysis in addition to flexure, and that conceptual subdivision of the section in the form of a square be utilized in place of a

horizontal layer, to allow for biaxial analysis. Moreover, provision should be made for the approach to take into account the slippage at interfaces of different materials, to enhance the capability of the approach for design of shear-connectors of a composite beam.

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