

PRODUK GRAF FUZZY INTUITIONISTIC

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Abstract: An intuitionistic fuzzy graph $G: \langle V, E \rangle$ consist of a couples of node sets V and set of edges E which the sum of degree membership and degree non membership each of nodes and each of edges in closed interval $[0,1]$, the degree membership each of edges is less than or equal with the minimum of degree membership each nodes of incident, and degree non membership each of edges is less than or equal with the maximum degree non membership each nodes of incident. Product fuzzy graph was defined by Dr. V. Ramaswamy and Poornima B. Product fuzzy graph is fuzzy graph, where the degree membership each of edges is less than or equal with product of the between degree of membership each nodes of incident. In this paper we study the definitions of product intuitionistic fuzzy graph, product intuitionistic fuzzy graph complete, further discussion about the complement of product intuitionistic fuzzy graph, join of product intuitionistic fuzzy graph and multiplication of product intuitionistic fuzzy graph and their characteristics.

Key words : intuitionistic fuzzy graph, product fuzzy graph, product intuitionistic fuzzy graph.

I. PENDAHULUAN

Pada tahun 1964 Lotfi Asker Zadeh mempublikasikan karangannya yang berjudul *fuzzy set* [5]. Dalam karangannya tersebut L.A. Zadeh memperluas konsep himpunan tegas yang merupakan kejadian khusus dari himpunan *fuzzy*. Salah satu bidang pembahasan tentang himpunan *fuzzy* yang terus berkembang pesat sampai sekarang adalah graf *fuzzy* yang diperkenalkan pertama kali oleh Rosenfield pada tahun 1975 [1]. R. Parvanthy dan M.G. Karunambigai memperkenalkan konsep tentang graf *fuzzy intuitionistic* dan beberapa komponennya. Suatu graf *fuzzy intuitionistic* dinotasikan dengan $G: \langle V, E \rangle$ dimana $V(G)$ adalah himpunan titik dan $E(G)$ adalah himpunan sisi.

II. DASAR TEORI

Definisi 2.1 [3]

Misalkan V adalah himpunan berhingga. Suatu graf *fuzzy* yang dinotasikan dengan $G = (V, \mu, \rho)$ adalah graf yang terdiri dari pasangan himpunan titik μ dan himpunan garis ρ dengan

- i. $\mu: V \rightarrow [0,1]$
- ii. $\rho: V \times V \rightarrow [0,1]$

yang memenuhi $\rho(xy) \leq \mu(x) \wedge \mu(y)$ untuk setiap $x, y \in V$.

Karena V himpunan berhingga (*finite*), maka penulisan graf *fuzzy* $G = (V, \mu, \rho)$ selanjutnya disederhanakan menjadi $G = (\mu, \rho)$.

Definisi 2.2 [2]

Sebuah Graf *Fuzzy intuitionistic* adalah suatu bentuk $G: \langle V, E \rangle$ dengan :

- (i) $V = \{v_1, v_2, \dots, v_n\}$ sedemikian sehingga $\mu_1: V \rightarrow [0,1]$ dan $\gamma_1: V \rightarrow [0,1]$ secara berturut-turut adalah derajat keanggotaan dan derajat bukan keanggotaan dari elemen $v_i \in V$, dan memenuhi $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$, untuk setiap $v_i \in V, (i = 1, 2, \dots, n)$.
- (ii) $E \subseteq V \times V$ dengan $\mu_2: V \times V \rightarrow [0,1]$ dan $\gamma_2: V \times V \rightarrow [0,1]$ yang memenuhi $\mu_2(v_i, v_j) \leq [\mu_1(v_i) \wedge \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \leq [\gamma_1(v_i) \wedge \gamma_1(v_j)]$ dan $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, untuk setiap $(v_i, v_j) \in E, (i, j = 1, 2, \dots, n)$.

Definisi 2.3 [4]

Misalkan $G^* = (V, E)$ adalah graf tegas dimana $E \subseteq V \times V$. Suatu produk parsial subgraf *fuzzy* dari $G^* = (V, E)$ yang dinotasikan dengan (μ, ρ) adalah graf yang terdiri dari pasangan himpunan titik μ dan himpunan garis ρ dengan,

- i. $\mu: V \rightarrow [0,1]$
- ii. $\rho: V \times V \rightarrow [0,1]$

yang memenuhi $\rho(xy) \leq \mu(x) \times \mu(y)$ untuk setiap $x, y \in V$, dimana untuk selanjutnya produk parsial subgraf *fuzzy* disingkat menjadi produk graf *fuzzy*.

Definisi 2.4 [4]

Misalkan (μ_1, ρ_1) dan (μ_2, ρ_2) berturut-turut adalah produk graf *fuzzy* dari $G_1^* = (V_1, E_1)$ dan $G_2^* = (V_2, E_2)$, maka perkalian dari (μ_1, ρ_1) dan (μ_2, ρ_2) yang dinotasikan $(\mu_1 \times \mu_2, \rho_1 \times \rho_2)$ didefinisikan sebagai berikut:

$(\mu_1 \times \mu_2)(v_1, v_2) = \mu_1(v_1) \times \mu_2(v_2)$ jika $v_1 \in V_1$ dan $v_2 \in V_2$
 $(\rho_1 \times \rho_2)((u_1, u_2)(v_1, v_2)) = \rho_1(u_1 v_1) \times \rho_2(u_2 v_2)$ untuk setiap $u_1, v_1 \in V_1$ dan untuk setiap $u_2, v_2 \in V_2$ dengan

- i. $\mu_1 \times \mu_2: V_1 \times V_2 \rightarrow [0,1]$
- ii. $\rho_1 \times \rho_2: (V_1 \times V_2) \times (V_1 \times V_2) \rightarrow [0,1]$

III. PEMBAHASAN

Definisi 3.1 [2]

Graf *fuzzy intuitionistic* $G: \langle V, E \rangle$ disebut produk parsial subgraf *fuzzy intuitionistic* jika memenuhi $\mu_2(x, y) \leq \mu_1(x) \times \mu_1(y)$ dan $\gamma_2(x, y) \leq \gamma_1(x) \times \gamma_1(y)$.

Untuk selanjutnya produk parsial subgraf *fuzzy intuitionistic* disingkat menjadi produk graf *fuzzy intuitionistic* dan dinotasikan dengan $G = \langle V, E \rangle$.

Contoh 3.1:

Diberikan graf *fuzzy* $G = (V, E)$ seperti pada Gambar 2.1, karena $\mu_1(v_1) \times \mu_1(v_2) = 0.3 \times 0.7 = 0.21 \geq 0.2 = \mu_2(v_1, v_2)$, $\gamma_1(v_1) \times \gamma_1(v_2) = 0.6 \times 0.2 = 0.12 \geq 0.1 = \gamma_2(v_1, v_2)$ maka graf *fuzzy* pada Gambar 2.1 merupakan produk graf *fuzzy intuitionistic*.

Teorema 3.2

Misalkan $G = \langle V, E \rangle$ adalah produk graf *fuzzy intuitionistic*, jika $0 \leq \mu_1(x) \leq 1$ dan $0 \leq \mu_1(y) \leq 1$, maka memenuhi $\mu_2(v_i, v_j) \leq \mu_1(v_i) \times \mu_1(v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$ dan $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \times \gamma_1(v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j)$ untuk semua $x, y \in V$. Sehingga produk graf *fuzzy intuitionistic* adalah graf *fuzzy intuitionistic*.

Bukti:

Misalkan $G = \langle V, E \rangle$ adalah produk graf *fuzzy intuitionistic*, jika $0 \leq \mu_1(x) \leq 1$ dan $0 \leq \mu_1(y) \leq 1$ untuk semua $x, y \in V$ didapatkan

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \times \mu_1(v_j) \quad (\text{Definisi 3.1}) \quad (1)$$

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j) \quad (\text{Definis 2.2 (ii)}) \quad (2)$$

Dari persamaan (1) dan (2) didapatkan

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \times \mu_1(v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j) \quad (3)$$

dan

$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \times \gamma_1(v_j) \quad (\text{Definisi 3.1}) \quad (4)$$

$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j) \quad (\text{Definisi 2.2(ii)}) \quad (5)$$

Dari persamaan (4) dan (5) didapatkan

$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \times \gamma_1(v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j) \quad (6)$$

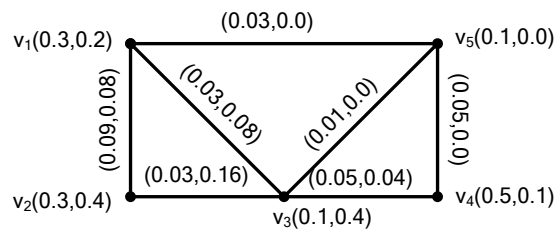
Tebukti bahwa produk graf *fuzzy intuitionistic* adalah graf *fuzzy intuitionistic*. ■

Definisi 3.3 [2]

Produk graf *fuzzy intuitionistic* $G = \langle V, E \rangle$ dikatakan lengkap jika $\mu_2(x, y) = \mu_1(x) \times \mu_1(y)$ dan $\gamma_2(x, y) = \gamma_1(x) \times \gamma_1(y)$ untuk semua $x, y \in V$.

Contoh 3.2 :

Diberikan produk graf *fuzzy intuitionistic* $G = \langle V, E \rangle$ pada Gambar 3.1 dimana $V = \{v_1, v_2, v_3, v_4, v_5\}$ dan $E = \{v_1v_2, v_1v_3, v_1v_5, v_2v_3, v_3v_4, v_3v_5, v_4v_5\}$ merupakan produk graf *fuzzy intuitionistic*



Gambar 3.1 Produk Graf *Fuzzy intuitionistic* $G = \langle V, E \rangle$

Teorema 3.4 [2]

$G = \langle V, E \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap dimana μ_1 dan γ_1 normal, maka $\mu_2^n(x, y) = \mu_2(x, y)$ dan $\gamma_2^n(x, y) = \gamma_2(x, y)$ untuk semua $x, y \in V$ dan untuk semua bilangan positif n untuk $n \geq 2$ berlaku

$$\mu_2^n(x, y) = \bigvee_{z \in V} \{ \mu_2^{n-1}(x, z) \times \mu_2(z, y) \}$$

$$\gamma_2^n(x, y) = \bigvee_{z \in V} \{ \gamma_2^{n-1}(x, z) \times \gamma_2(z, y) \}$$

Bukti :

untuk membuktikan Teorema 3.4 digunakan induksi matematika sebagai berikut:

a) Langkah pertama:

Jika $G = \langle V, E \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap, maka untuk $n = 2$ dan untuk semua $x, y \in V$ diperoleh

$$\begin{aligned} \mu_2^2(x, y) &= \bigvee_{z \in V} \{ \mu_2(x, z) \times \mu_2(z, y) \} \\ &= \bigvee_{z \in V} \{ [\mu_1(x) \times \mu_1(z)] \times [\mu_1(z) \times \mu_1(y)] \} \quad (\text{Definisi 3.3}) \\ &= \bigvee_{z \in V} \{ \mu_1(x) \times \mu_1(y) \times \mu_1^2(z) \} \end{aligned}$$

Karena $\mu_1^2(z) \leq 1$ untuk semua z , sehingga

$$\mu_2^2(x, y) = \bigvee_{z \in V} \{ \mu_1(x) \times \mu_1(y) \times \mu_1^2(z) \} \leq \mu_1(x) \times \mu_1(y)$$

$$\mu_2^2(x, y) \leq \mu_1(x) \times \mu_1(y) = \mu_2(x, y)$$

$$\mu_2^2(x, y) \leq \mu_2(x, y) \quad (\text{Definisi 3.3}) \quad (1)$$

dan

$$\begin{aligned}
\gamma_2^2(x, y) &= \bigvee_{z \in V} \{ \gamma_2(x, z) \times \gamma_2(z, y) \} \\
&= \bigvee_{z \in V} \{ [\gamma_1(x) \times \gamma_1(z)] \times [\gamma_1(z) \times \gamma_1(y)] \} \quad (\text{Definisi 3.3.1}) \\
&= \bigvee_{z \in V} \{ \gamma_1(x) \times \gamma_1(y) \times \gamma_1^2(z) \}
\end{aligned}$$

Karena $\gamma_1^2(z) \leq 1$ untuk semua z , sehingga

$$\begin{aligned}
\gamma_2^2(x, y) &= \bigvee_{z \in V} \{ \gamma_1(x) \times \gamma_1(y) \times \gamma_1^2(z) \} \leq \gamma_1(x) \times \gamma_1(y) \\
\gamma_2^2(x, y) &\leq \gamma_1(x) \times \gamma_1(y) = \gamma_2(x, y) \\
\gamma_2^2(x, y) &\leq \gamma_2(x, y) \quad (\text{Definisi 3.3}) \quad (2)
\end{aligned}$$

Jika μ_1 dan γ_1 normal, maka $\mu_1(t) = 1$ dan $\gamma_1(t) = 1$ untuk suatu t , sehingga

$$\begin{aligned}
\mu_2^2(x, y) &= \bigvee_{z \in V} \{ \mu_1(x) \times \mu_1(y) \times \mu_1^2(z) \} \\
&\geq \mu_1(x) \times \mu_1(y) \times \mu_1^2(t) \quad (\text{Definisi sup } \forall) \\
&= \mu_1(x) \times \mu_1(y) \quad (\text{karena } \mu_1(t) = 1)
\end{aligned}$$

$$\mu_2^2(x, y) \geq \mu_1(x) \times \mu_1(y)$$

$$\mu_2^2(x, y) \geq \mu_2(x, y) \quad (3)$$

karena $\mu_2^2(x, y) = \mu_1(x) \times \mu_1(y)$ maka G lengkap

Dari persamaan (1) dan (3) didapatkan

$$\mu_2^2(x, y) = \mu_2(x, y) \quad (4)$$

dan

$$\begin{aligned}
\gamma_2^2(x, y) &= \bigvee_{z \in V} \{ \gamma_1(x) \times \gamma_1(y) \times \gamma_1^2(z) \} \\
&\geq \gamma_1(x) \times \gamma_1(y) \times \gamma_1^2(t) \quad (\text{Definisi sup } \forall) \\
&= \gamma_1(x) \times \gamma_1(y) \quad (\text{karena } \gamma_1(t) = 1)
\end{aligned}$$

$$\gamma_2^2(x, y) \geq \gamma_1(x) \times \gamma_1(y)$$

$$\gamma_2^2(x, y) \geq \gamma_2(x, y) \quad (5)$$

karena $\gamma_2^2(x, y) = \gamma_1(x) \times \gamma_1(y)$ maka G lengkap

Dari persamaan (2) dan (5), didapatkan

$$\gamma_2^2(x, y) = \gamma_2(x, y) \quad (6)$$

b) Langkah Induksi:

Sekarang diasumsikan bahwa

$$\mu_2^k(x, y) = \mu_2(x, y) \text{ dan } \gamma_2^k(x, y) = \gamma_2(x, y) \quad (7)$$

Akan dibuktikan bahwa $\mu_2^{k+1}(x, y) = \mu_2(x, y)$ dan $\gamma_2^{k+1}(x, y) = \gamma_2(x, y)$

didapatkan:

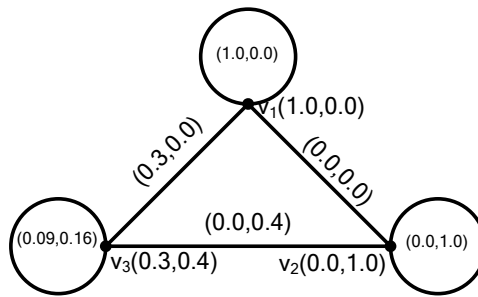
$$\begin{aligned}
\mu_2^{k+1}(x, y) &= \bigvee_{z \in V} \{ \mu_2^k(x, z) \times \mu_2(z, y) \} \\
&= \bigvee_{z \in V} \{ \mu_2(x, z) \times \mu_2(z, y) \} \quad (\text{asumsi (7)})
\end{aligned}$$

$$\begin{aligned}
&= \mu_2^2(x, y) && \text{(langkah pertama untuk } n = 2) \\
\mu_2^{k+1}(x, y) &= \mu_2(x, y) \\
\text{dan} \\
\gamma_2^{k+1}(x, y) &= \bigvee_{z \in V} \{ \gamma_2^k(x, z) \times \gamma_2(z, y) \} \\
&= \bigvee_{z \in V} \{ \gamma_2(x, z) \times \gamma_2(z, y) \} && \text{(asumsi (7))} \\
&= \gamma_2^2(x, y) && \text{(langkah pertama untuk } n = 2) \\
\gamma_2^{k+1}(x, y) &= \gamma_2(x, y)
\end{aligned}$$

Dari penyelesaian a) dan b) maka dapat ditunjukkan bahwa Teorema 3.4 Terbukti. ■

Contoh 3.3 :

Misalkan diberikan graf *fuzzy intuitionistic* $G: \langle V, E \rangle$ dengan μ_1 dan γ_1 normal pada Gambar 3.2 dimana $V = \{v_1, v_2, v_3\}$ dan $E = \{v_1v_1, v_1v_2, v_1v_3, v_2v_2, v_2v_3, v_3v_3\}$ merupakan produk graf *fuzzy intuitionistic* lengkap



Gambar 3.2 Graf *fuzzy intuitionistic* $G: \langle V, E \rangle$

$$\begin{aligned}
\mu_2^2(v_1, v_2) &= [\mu_2(v_1, v_1) \times \mu_2(v_1, v_2)] \vee [\mu_2(v_1, v_2) \times \mu_2(v_2, v_2)] \vee \\
&\quad [\mu_2(v_1, v_3) \times \mu_2(v_3, v_2)] \\
&= (1.0 \times 0.0) \vee (0.0 \times 0.0) \vee (0.3 \times 0.0) = 0.0
\end{aligned}$$

Karena diketahui $\mu_2(v_1, v_2) = 0.0$, sehingga $\mu_2^2(v_1, v_2) = \mu_2(v_1, v_2)$.

$$\begin{aligned}
\gamma_2^2(v_1, v_2) &= [\gamma_2(v_1, v_1) \times \gamma_2(v_1, v_2)] \vee [\gamma_2(v_1, v_2) \times \gamma_2(v_2, v_2)] \vee \\
&\quad [\gamma_2(v_1, v_3) \times \gamma_2(v_3, v_2)] \\
&= (0.0 \times 0.0) \vee (0.0 \times 1.0) \vee (0.0 \times 0.4) = 0.0
\end{aligned}$$

Karena diketahui $\gamma_2(v_1, v_2) = 0.0$, sehingga $\gamma_2^2(v_1, v_2) = \gamma_2(v_1, v_2)$

Definisi 3.5 [2]

Komplemen dari produk graf *fuzzy intuitionistic* $G = \langle V, E \rangle$ adalah $G^c = \langle V^c, E^c \rangle$ dimana $V^c = (\mathbb{Q}_1^c, \gamma_1^c)$ dan $E^c = (\mathbb{Q}_2^c, \gamma_2^c)$ dengan $\mathbb{Q}_1^c = \mu_1, \gamma_1^c = \gamma_1$ dan

$$\begin{aligned}\mathbb{Q}_2^c(x, y) &= \mathbb{Q}_1(x) \times \mathbb{Q}_1(y) - \mathbb{Q}_2(x, y), \\ \gamma_2^c(x, y) &= \gamma_1(x) \times \gamma_1(y) - \gamma_2(x, y)\end{aligned}$$

Catatan:

Doble komplemen dari produk graf *fuzzy intuitionistic* yang dinotasikan dengan $(G^c)^c = \langle (V^c)^c, (E^c)^c \rangle$ menghasilkan $G = \langle V, E \rangle$

Definisi 3.6 [2]

Jika $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ adalah produk graf *fuzzy intuitionistic*. Disini $V_1 = (\mathbb{Q}_{11}, \gamma_{11})$, $E_1 = (\mathbb{Q}_{12}, \gamma_{12})$, $V_2 = (\mathbb{Q}_{21}, \gamma_{21})$, dan $E_2 = (\mathbb{Q}_{22}, \gamma_{22})$. Jika X' menunjukkan himpunan semua garis yang menghubungkan beberapa titik V_1 ke titik V_2 . Lebih lanjut akan mengasumsikan $V_1 \cap V_2 = \emptyset$, maka join dari G_1 dan G_2 didefinisikan sebagai $G(V_1 + V_2, E_1 + E_2)$ dimana $V_1 + V_2 = (\mathbb{Q}_{11} + \mathbb{Q}_{21}, \gamma_{11} + \gamma_{21})$ dan $E_1 + E_2 = (\mathbb{Q}_{12} + \mathbb{Q}_{22}, \gamma_{12} + \gamma_{22})$ disini

$$\begin{aligned}(\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) &= \mathbb{Q}_{11}(u) \text{ jika } u \in V_1 \\ &= \mathbb{Q}_{21}(u) \text{ jika } u \in V_2 \\ (\gamma_{11} + \gamma_{21})(u) &= \gamma_{11}(u) \text{ jika } u \in V_1 \\ &= \gamma_{21}(u) \text{ jika } u \in V_2 \\ (\mathbb{Q}_{12} + \mathbb{Q}_{22})(u, v) &= \mathbb{Q}_{12}(u, v) \text{ jika } (u, v) \in E_1 \\ &= \mathbb{Q}_{22}(u, v) \text{ jika } (u, v) \in E_2 \\ &= \mu_{11}(u) \times \mu_{21}(v) \text{ jika } (u, v) \in X' \\ (\gamma_{12} + \gamma_{22})(u, v) &= \gamma_{12}(u, v) \text{ jika } (u, v) \in E_1 \\ &= \gamma_{22}(u, v) \text{ jika } (u, v) \in E_2 \\ &= \gamma_{11}(u) \times \gamma_{21}(v) \text{ jika } (u, v) \in X'\end{aligned}$$

Teorema 3.7 [2]

Misalkan $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ berturut-turut adalah produk graf *fuzzy intuitionistic*, maka $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* dimana $E = E_1 \cup E_2 \cup X'$.

Bukti:

untuk membuktikan bahwa $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* dimana $E = E_1 \cup E_2 \cup X'$ yaitu dengan membuktikan bahwa

$$(\mu_{12} + \mu_{22})(u, v) \leq (\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{12})(v) \tag{1}$$

$$\text{dan } (\gamma_{12} + \gamma_{22})(u, v) \leq (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{12})(v) \tag{2}$$

untuk semua $(u, v) \in V$ sebagai berikut :

a) Jika $(u, v) \in E_1$, maka $u, v \in V_1$, sehingga

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{12}(u, v) \quad (\text{Definisi 3.6}) \quad (3)$$

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{12})(v) = \mu_{11}(u) \times \mu_{11}(v) \quad (\text{Definisi 3.6}) \quad (4)$$

Dari persamaan (3) dan (4) memenuhi persamaan (1).

dan

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{12}(u, v) \quad (\text{Definisi 3.6}) \quad (5)$$

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{12})(v) = \gamma_{11}(u) \times \gamma_{11}(v) \quad (\text{Definisi 3.6}) \quad (6)$$

Dari persamaan (5) dan (6) memenuhi persamaan (2).

b) Jika $(u, v) \in X'$, maka $u \in V_1$ dan $v \in V_2$, sehingga

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{11}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6}) \quad (7)$$

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{11}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6}) \quad (8)$$

Dari persamaan (7) dan (8) didapatkan

$$(\mu_{12} + \mu_{22})(u, v) = (\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) \quad (\text{Definisi 3.6}) \quad (9)$$

dan

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{11}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6}) \quad (10)$$

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{11}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6}) \quad (11)$$

Dari persamaan (10) dan (11) didapatkan

$$(\gamma_{12} + \gamma_{22})(u, v) = (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) \quad (\text{Definisi 3.6}) \quad (12)$$

Dari (a) dan (b) membuktikan bahwa $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* dimana $E = E_1 \cup E_2 \cup X'$. ■

Teorema 3.8 [2]

Misalkan $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ berturut-turut adalah produk graf *fuzzy intuitionistic* lengkap, maka $G(V_1 + V_2, E_1 + E_2)$ adalah lengkap jika hanya jika $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ keduanya adalah produk graf *fuzzy intuitionistic* lengkap.

Bukti:

(\leftarrow) Pertama akan diasumsikan bahwa $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ keduanya adalah produk graf *fuzzy intuitionistic* lengkap, maka akan dibuktikan bahwa $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* lengkap.

a) Jika $(u, v) \in E_1$ maka $u, v \in V_1$ sehingga

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{12}(u, v) = \mu_{11}(u) \times \mu_{11}(v) \quad (\text{Definisi 3.6})$$

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{11}(u) \times \mu_{11}(v)$$

dan

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{12}(u, v) = \gamma_{11}(u) \times \gamma_{11}(v) \quad (\text{Definisi 3.6})$$

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{11}(u) \times \gamma_{11}(v)$$

b) Jika $(u, v) \in E_2$, maka $u, v \in V_2$ sehingga

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{22}(u, v) = \mu_{21}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6})$$

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{21}(u) \times \mu_{21}(v)$$

dan

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{22}(u, v) = \gamma_{21}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6})$$

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{21}(u) \times \gamma_{21}(v)$$

c) Jika $(u, v) \in X'$, maka $u \in V_1$ dan $v \in V_2$ sehingga

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{11}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6}) (1)$$

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{11}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6}) (2)$$

Dari persamaan (1) disubstitusikan ke persamaan (2) didapatkan

$$(\mu_{12} + \mu_{22})(u, v) = (\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v)$$

dan

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{11}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6}) (3)$$

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{11}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6}) (4)$$

Dari persamaan (3) disubstitusikan ke persamaan (4) didapatkan

$$(\gamma_{12} + \gamma_{22})(u, v) = (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v)$$

Dari (a), (b), dan (c) terbukti bahwa $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* lengkap.

(\rightarrow) Sebaliknya diasumsikan bahwa $G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* lengkap maka akan dibuktikan $G_1 = \langle V_1, E_1 \rangle$ dan $G_2 = \langle V_2, E_2 \rangle$ keduanya adalah produk graf *fuzzy intuitionistic* lengkap.

a) $G_1 = \langle V_1, E_1 \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap, maka akan dibuktikan bahwa

$$\mu_{12}(u, v) = \mu_{11}(u) \times \mu_{11}(v) \quad (\text{Definisi 3.3}) (5)$$

$$\text{dan } \gamma_{12}(u, v) = \gamma_{11}(u) \times \gamma_{11}(v) \quad (\text{Definisi 3.3}) (6)$$

untuk semua $(u, v) \in E_1$

Jika $(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* lengkap, maka

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{12}(u, v) \quad (\text{Definisi 3.6}) (7)$$

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{12}(u, v) \quad (\text{Definisi 3.6}) \quad (8)$$

Jika $u, v \in E_1$ maka $u, v \in V_1$, sehingga

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{11}(u) \times \mu_{11}(v) \quad (\text{Definisi 3.6}) \quad (9)$$

$$(\mu_{12} + \mu_{22})(u, v) = (\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) \quad (\text{Definisi 3.6}) \quad (10)$$

dan

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{11}(u) \times \gamma_{11}(v) \quad (\text{Definisi 3.6}) \quad (11)$$

$$(\gamma_{12} + \gamma_{22})(u, v) = (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) \quad (\text{Definisi 3.6}) \quad (12)$$

Dari persamaan (7) dan (9) didapatkan persamaan (5) yaitu

$$\mu_{12}(u, v) = \mu_{11}(u) \times \mu_{11}(v)$$

Dari persamaan (8) dan (11) didapatkan persamaan (6) yaitu

$$\gamma_{12}(u, v) = \gamma_{11}(u) \times \gamma_{11}(v)$$

maka $G_1 = \langle V_1, E_1 \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap.

- b) $G_2 = \langle V_2, E_2 \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap, maka akan dibuktikan bahwa

$$\mu_{22}(u, v) = \mu_{21}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.3}) \quad (13)$$

$$\text{dan } \gamma_{22}(u, v) = \gamma_{21}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.3}) \quad (14)$$

untuk semua $(u, v) \in E_2$

$G(V_1 + V_2, E_1 + E_2)$ adalah produk graf *fuzzy intuitionistic* lengkap maka

$$(\mu_{12} + \mu_{22})(u, v) = \mu_{22}(u, v) \quad (\text{Definisi 3.6}) \quad (15)$$

$$(\gamma_{12} + \gamma_{22})(u, v) = \gamma_{22}(u, v) \quad (\text{Definisi 3.6}) \quad (16)$$

Jika $u, v \in E_2$ maka $u, v \in V_2$ sehingga

$$(\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) = \mu_{21}(u) \times \mu_{21}(v) \quad (\text{Definisi 3.6}) \quad (17)$$

$$(\mu_{12} + \mu_{22})(u, v) = (\mu_{11} + \mu_{21})(u) \times (\mu_{11} + \mu_{21})(v) \quad (\text{Definisi 3.6}) \quad (18)$$

dan

$$(\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) = \gamma_{21}(u) \times \gamma_{21}(v) \quad (\text{Definisi 3.6}) \quad (19)$$

$$(\gamma_{12} + \gamma_{22})(u, v) = (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) \quad (\text{Definisi 3.6}) \quad (20)$$

Dari persamaan (15) dan (17) didapatkan persamaan (13) yaitu

$$\mu_{22}(u, v) = \mu_{21}(u) \times \mu_{21}(v)$$

Dari persamaan (16) dan (18) didapatkan persamaan (14) yaitu

$$\gamma_{22}(u, v) = \gamma_{21}(u) \times \gamma_{21}(v)$$

maka $G_2 = \langle V_2, E_2 \rangle$ adalah produk graf *fuzzy intuitionistic* lengkap.

Jadi Teorema 3.5.2 terbukti. ■

Teorema 3.9 [2]

Jika G_1 dan G_2 adalah produk graf *fuzzy intuitionistic*, maka sifat berikut terpenuhi:

- i. $(\mathbb{Q}_{11} + \mathbb{Q}_{21}, \mathbb{Q}_{12} + \mathbb{Q}_{22})^c = (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c, \mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)$
- ii. $(\gamma_{11} + \gamma_{21}, \gamma_{12} + \gamma_{22})^c = (\gamma_{11}^c \cup \gamma_{21}^c, \gamma_{12}^c \cup \gamma_{22}^c)$

Bukti:

a. Jika $u \in V_1$ maka

$$(\mathbb{Q}_{11} + \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) \quad (\text{Definisi 3.5})$$

$$= \mathbb{Q}_{11}(u) \quad (\text{Definisi 3.6})$$

$$= \mathbb{Q}_{11}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c)(u)$$

$$\max(\mathbb{Q}_{11}^c(u), \mathbb{Q}_{21}^c(u)) = \max(\mathbb{Q}_{11}(u), \mathbb{Q}_{21}(u)) \quad (\text{Definisi 3.5})$$

$$= \mathbb{Q}_{11}(u) \quad (\text{Definisi 3.6})$$

$$= \mathbb{Q}_{11}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c)(u)$$

dan

$$(\gamma_{11} + \gamma_{21})^c(u) = (\gamma_{11} + \gamma_{21})(u) \quad (\text{Definisi 3.5})$$

$$= \gamma_{11}(u) \quad (\text{Definisi 3.6})$$

$$= \gamma_{11}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{11}^c \cup \gamma_{21}^c)(u)$$

$$\min(\gamma_{11}^c(u), \gamma_{21}^c(u)) = \min(\gamma_{11}(u), \gamma_{21}(u)) \quad (\text{Definisi 3.6})$$

$$= \gamma_{11}(u)$$

$$= \gamma_{11}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{11}^c \cup \gamma_{21}^c)(u)$$

b. Jika $u \in V_2$ maka

$$(\mathbb{Q}_{11} + \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) \quad (\text{Definisi 3.5})$$

$$= \mathbb{Q}_{21}(u) \quad (\text{Definisi 3.6})$$

$$= \mathbb{Q}_{21}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c)(u)$$

$$\max(\mathbb{Q}_{11}^c(u), \mathbb{Q}_{21}^c(u)) = \max(\mathbb{Q}_{11}(u), \mathbb{Q}_{21}(u)) \\ = \mathbb{Q}_{21}(u) \quad (\text{Definisi 3.6})$$

$$= \mathbb{Q}_{21}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c)(u)$$

dan

$$(\gamma_{11} + \gamma_{21})^c(u) = (\gamma_{11} + \gamma_{21})(u) \quad (\text{Definisi 3.5})$$

$$= \gamma_{21}(u) \quad (\text{Definisi 3.6})$$

$$= \gamma_{21}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{11}^c \cup \gamma_{21}^c)(u)$$

$$\min(\gamma_{11}^c(u), \gamma_{21}^c(u)) = \min(\gamma_{11}(u), \gamma_{21}(u)) \\ = \gamma_{21}(u) \quad (\text{Definisi 3.6})$$

$$= \gamma_{21}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{11}^c \cup \gamma_{21}^c)(u)$$

Dari (a) dan (b) membuktikan bahwa

$$(\mathbb{Q}_{11} + \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c)(u) \quad (1)$$

$$(\gamma_{11} + \gamma_{21})^c(u) = (\gamma_{11}^c \cup \gamma_{21}^c)(u) \quad (2)$$

c. Misalkan $(u, v) \in E_1$ dan $u, v \in V_1$, maka berdasarkan Definisi 3.5 didapatkan

$$\begin{aligned}
& (\mathbb{Q}_{12} + \mathbb{Q}_{22})^c(u, v) \\
&= (\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} + \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} + \mathbb{Q}_{22})(u, v) \\
&= \mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) - \mathbb{Q}_{12}(u, v) && \text{(Definisi 3.6)} \\
&= \mathbb{Q}_{12}^c(u, v) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v)
\end{aligned}$$

$$\begin{aligned}
\max(\mathbb{Q}_{12}^c(u, v), \mathbb{Q}_{22}^c(u, v)) &= \max(\mathbb{Q}_{12}(u, v), \mathbb{Q}_{22}(u, v)) \\
&= \mathbb{Q}_{12}(u, v) && \text{(Definisi 3.6)} \\
&= \mathbb{Q}_{12}^c(u, v) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v)
\end{aligned}$$

dan

$$\begin{aligned}
& (\gamma_{12} + \gamma_{22})^c(u, v) \\
&= (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) - (\gamma_{12} + \gamma_{22})(u, v) \\
&= \gamma_{11}(u) \times \gamma_{11}(v) - \gamma_{12}(u, v) && \text{(Definisi 3.6)} \\
&= \gamma_{12}^c(u, v) && \text{(Definisi 3.5)} \\
&= (\gamma_{12}^c \cup \gamma_{22}^c)(u, v)
\end{aligned}$$

$$\begin{aligned}
\min(\gamma_{12}^c(u, v), \gamma_{22}^c(u, v)) &= \min(\gamma_{12}(u, v), \gamma_{22}(u, v)) \\
&= \gamma_{12}(u, v) && \text{(Definisi 3.6)} \\
&= \gamma_{12}^c(u, v) && \text{(Definisi 3.5)} \\
&= (\gamma_{12}^c \cup \gamma_{22}^c)(u, v)
\end{aligned}$$

d. Misalkan $(u, v) \in E_2$ dan $u, v \in V_2$, maka berdasarkan Definisi 3.5 didapatkan

$$\begin{aligned}
& (\mathbb{Q}_{12} + \mathbb{Q}_{22})^c(u, v) \\
&= (\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} + \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} + \mathbb{Q}_{22})(u, v) \\
&= \mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) - \mathbb{Q}_{12}(u, v) && \text{(Definisi 3.6)} \\
&= \mathbb{Q}_{12}^c(u, v) && \text{(Definisi 3.5)}
\end{aligned}$$

$$= (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v)$$

$$\max(\mathbb{Q}_{12}^c(u, v), \mathbb{Q}_{22}^c(u, v)) = \max(\mathbb{Q}_{12}(u, v), \mathbb{Q}_{22}(u, v))$$

$$= \mathbb{Q}_{12}(u, v) \quad (\text{Definisi 3.6})$$

$$= \mathbb{Q}_{12}^c(u, v) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v)$$

dan

$$(\gamma_{12} + \gamma_{22})^c(u, v)$$

$$= (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) - (\gamma_{12} + \gamma_{22})(u, v)$$

$$= \gamma_{11}(u) \times \gamma_{11}(v) - \gamma_{12}(u, v) \quad (\text{Definisi 3.6})$$

$$= \gamma_{12}^c(u, v) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{12}^c \cup \gamma_{22}^c)(u, v)$$

$$\min(\gamma_{12}^c(u, v), \gamma_{22}^c(u, v)) = \min(\gamma_{12}(u, v), \gamma_{22}(u, v))$$

$$= \gamma_{12}(u, v) \quad (\text{Definisi 3.6})$$

$$= \gamma_{12}^c(u, v) \quad (\text{Definisi 3.5})$$

$$= (\gamma_{12}^c \cup \gamma_{22}^c)(u, v)$$

e. Jika $(u, v) \in X'$, maka $u \in V_1$ dan $v \in V_2$

$$(\mathbb{Q}_{12} + \mathbb{Q}_{22})^c(u, v)$$

$$= (\mathbb{Q}_{11} + \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} + \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} + \mathbb{Q}_{22})(u, v)$$

$$= \mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) - \mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) \quad (\text{Definisi 3.6})$$

$$= 0$$

$$= \max(0, 0)$$

$$= \max(\mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) - \mathbb{Q}_{12}(u, v), \mathbb{Q}_{21}(u) \times \mathbb{Q}_{21}(v) - \mathbb{Q}_{22}(u, v))^*$$

$$= \max(\mathbb{Q}_{12}^c(u, v), \mathbb{Q}_{22}^c(u, v))$$

$$= (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v)$$

Catatan* : $\mathbb{Q}_{11}(u) = 0, \mathbb{Q}_{12}(u) = 0, \mathbb{Q}_{11}(u, v) = 0, \mathbb{Q}_{12}(u, v) = 0$

dan

$$\begin{aligned}
 & (\gamma_{12} + \gamma_{22})^c(u, v) \\
 &= (\gamma_{11} + \gamma_{21})(u) \times (\gamma_{11} + \gamma_{21})(v) - (\gamma_{12} + \gamma_{22})(u, v) \\
 &= \gamma_{11}(u) \times \gamma_{11}(v) - \gamma_{11}(u) \times \gamma_{11}(v) \quad (\text{Definisi 3.6}) \\
 &= 0 \\
 &= \min(\mathbb{Q}, 0) \\
 &= \min(\gamma_{11}(u) \times \gamma_{11}(v) - \gamma_{12}(u, v), \gamma_{21}(u) \times \gamma_{21}(v) - \gamma_{22}(u, v))^* \\
 &= \min(\gamma_{12}^c(u, v), \gamma_{22}^c(u, v)) \\
 &= (\gamma_{12}^c \cup \gamma_{22}^c)(u, v)
 \end{aligned}$$

Catatan* : $\gamma_{11}(u) = 0, \gamma_{21}(u) = 0, \gamma_{12}(u, v) = 0, \gamma_{22}(u, v) = 0$

Dari (c), (d) dan (e) membuktikan bahwa

$$(\mathbb{Q}_{12} + \mathbb{Q}_{22})^c(u, v) = (\mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c)(u, v) \quad (3)$$

$$(\gamma_{12} + \gamma_{22})^c(u, v) = (\gamma_{12}^c \cup \gamma_{22}^c)(u, v) \quad (4)$$

Jadi dari (1), (2), (3) dan (4) dapat dibuktikan bahwa

$$(\mathbb{Q}_{11} + \mathbb{Q}_{21}, \mathbb{Q}_{12} + \mathbb{Q}_{22})^c = (\mathbb{Q}_{11}^c \cup \mathbb{Q}_{21}^c, \mathbb{Q}_{12}^c \cup \mathbb{Q}_{22}^c) \text{ dan}$$

$$(\gamma_{11} + \gamma_{21}, \gamma_{12} + \gamma_{22})^c = (\gamma_{11}^c \cup \gamma_{21}^c, \gamma_{12}^c \cup \gamma_{22}^c) \quad \blacksquare$$

Teorema 3.10 [2]

Jika G_1 dan G_2 adalah produk graf *fuzzy intuitionistic*, maka

$$((\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c, (\mathbb{Q}_{12} \cup \mathbb{Q}_{22})^c) = (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c, \mathbb{Q}_{12}^c + \mathbb{Q}_{22}^c)$$

$$((\gamma_{11} \cup \gamma_{21})^c, (\gamma_{12} \cup \gamma_{22})^c) = (\gamma_{11}^c + \gamma_{21}^c, \gamma_{12}^c + \gamma_{22}^c)$$

Bukti:

a. Jika $u \in V_1$, maka

$$(\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(u) \quad (\text{Definisi 3.5})$$

$$= \mathbb{Q}_{11}(u)$$

$$= \mathbb{Q}_{11}^c(u) \quad (\text{Definisi 3.5})$$

$$= (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u) \quad (\text{Definisi 3.6}) \quad (1)$$

$$\max(\mathbb{Q}_{11}(u)^c, \mathbb{Q}_{21}(u)^c) = \max(\mathbb{Q}_{11}(u), \mathbb{Q}_{21}(u))$$

$$\begin{aligned}
&= \mathbb{Q}_{11}(u) && \text{(Definisi 3.6)} \\
&= \mathbb{Q}_{11}^c(u) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u) && \text{(Definisi 3.6) (2)}
\end{aligned}$$

Dari persamaan (1) dan (2) didapatkan

$$(\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u)$$

dan

$$\begin{aligned}
(\gamma_{11} \cup \gamma_{21})^c(u) &= (\gamma_{11} \cup \gamma_{21})(u) && \text{(Definisi 3.5)} \\
&= \gamma_{11}(u) \\
&= \gamma_{11}^c(u) && \text{(Definisi 3.5)} \\
&= (\gamma_{11}^c + \gamma_{21}^c)(u) && \text{(Definisi 3.6) (3)}
\end{aligned}$$

$$\begin{aligned}
\min(\gamma_{11}(u)^c, \gamma_{21}(u)^c) &= \min(\gamma_{11}(u), \gamma_{21}(u)) && \text{(Definisi 3.6)} \\
&= \gamma_{11}(u) && \text{(Definisi 3.5)} \\
&= \gamma_{11}^c(u) && \text{(Definisi 3.5)} \\
&= (\gamma_{11}^c + \gamma_{21}^c)(u) && \text{(Definisi 3.6) (4)}
\end{aligned}$$

Dari persamaan (3) dan (4) didapatkan

$$(\gamma_{11} \cup \gamma_{21})^c(u) = (\gamma_{11}^c + \gamma_{21}^c)(u)$$

b. Jika $u \in V_2$, maka

$$\begin{aligned}
(\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c(u) &= (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(u) && \text{(Definisi 3.5)} \\
&= \mathbb{Q}_{21}(u) \\
&= \mathbb{Q}_{21}^c(u) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u) && \text{(Definisi 3.6) (5)}
\end{aligned}$$

$$\begin{aligned}
\max(\mathbb{Q}_{11}(u)^c, \mathbb{Q}_{21}(u)^c) &= \max(\mathbb{Q}_{11}(u), \mathbb{Q}_{21}(u)) && \text{(Definisi 3.6)} \\
&= \mathbb{Q}_{21}(u) && \text{(Definisi 3.5)} \\
&= \mathbb{Q}_{21}^c(u) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u) && \text{(Definisi 3.6) (6)}
\end{aligned}$$

Dari persamaan (5) dan (6) didapatkan

$$(\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c(u) = (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c)(u)$$

dan

$$\begin{aligned}
(\gamma_{11} \cup \gamma_{21})^c(u) &= (\gamma_{11} \cup \gamma_{21})(u) && \text{(Definisi 3.5)} \\
&= \gamma_{21}(u) \\
&= \gamma_{21}^c(u) && \text{(Definisi 3.5)} \\
&= (\gamma_{11}^c + \gamma_{21}^c)(u) && \text{(Definisi 3.6) (7)}
\end{aligned}$$

$$\min(\gamma_{11}(u)^c, \gamma_{21}(u)^c) = \min(\gamma_{11}(u), \gamma_{21}(u))$$

$$\begin{aligned}
&= \gamma_{21}(u) && \text{(Definisi 3.6)} \\
&= \gamma_{21}^c(u) && \text{(Definisi 3.5)} \\
&= (\gamma_{11}^c + \gamma_{21}^c)(u) && \text{(Definisi 3.6) (8)}
\end{aligned}$$

Dari persamaan (7) dan (8) didapatkan

$$(\gamma_{11} \cup \gamma_{21})^c(u) = (\gamma_{11}^c + \gamma_{21}^c)(u)$$

c. Jika $(u, v) \in E_1$, maka $u, v \in V_1$, sehingga

$$\begin{aligned}
&(\mathbb{Q}_{12} \cup \mathbb{Q}_{22})^c(u, v) \\
&= (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} \cup \mathbb{Q}_{22})(u, v) \\
&= \mathbb{Q}_{11}(u) \times \mathbb{Q}_{11}(v) - \mathbb{Q}_{12}(u, v) \\
&= \mathbb{Q}_{12}^c(u, v) && \text{(Definisi 3.5)}
\end{aligned}$$

dan

$$\begin{aligned}
&(\gamma_{12} \cup \gamma_{22})^c(u, v) \\
&= (\gamma_{11} \cup \gamma_{21})(u) \times (\gamma_{11} \cup \gamma_{21})(v) - (\gamma_{12} \cup \gamma_{22})(u, v) \\
&= \gamma_{11}(u) \times \gamma_{11}(v) - \gamma_{12}(u, v) \\
&= \gamma_{12}^c(u, v) && \text{(Definisi 3.5)}
\end{aligned}$$

d. Jika $(u, v) \in E_2$, maka $u, v \in V_2$, sehingga

$$\begin{aligned}
&(\mathbb{Q}_{12} \cup \mathbb{Q}_{22})^c(u, v) \\
&= (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} \cup \mathbb{Q}_{22})(u, v) \\
&= \mathbb{Q}_{21}(u) \times \mathbb{Q}_{21}(v) - \mathbb{Q}_{22}(u, v) \\
&= \mathbb{Q}_{22}^c(u, v) && \text{(Definisi 3.5)}
\end{aligned}$$

dan

$$\begin{aligned}
&(\gamma_{12} \cup \gamma_{22})^c(u, v) \\
&= (\gamma_{11} \cup \gamma_{21})(u) \times (\gamma_{11} \cup \gamma_{21})(v) - (\gamma_{12} \cup \gamma_{22})(u, v) \\
&= \gamma_{21}(u) \times \gamma_{21}(v) - \gamma_{22}(u, v) \\
&= \gamma_{22}^c(u, v) && \text{(Definisi 3.5)}
\end{aligned}$$

e. Jika $(u, v) \in X'$, maka $u \in V_1$ dan $v \in V_2$, sehingga

$$\begin{aligned}
&(\mathbb{Q}_{12} \cup \mathbb{Q}_{22})^c(u, v) \\
&= (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(u) \times (\mathbb{Q}_{11} \cup \mathbb{Q}_{21})(v) - (\mathbb{Q}_{12} \cup \mathbb{Q}_{22})(u, v) \\
&= \mathbb{Q}_{11}(u) \times \mathbb{Q}_{21}(v) \\
&\quad \text{karena } \mathbb{Q}_{12}(u, v) = \mathbb{Q}_{22}(u, v) = 0 \\
&= \mathbb{Q}_{11}^c(u) \times \mathbb{Q}_{21}^c(v) && \text{(Definisi 3.5)} \\
&= (\mathbb{Q}_{12}^c + \mathbb{Q}_{22}^c)(u, v) && \text{(Definisi 3.6)}
\end{aligned}$$

dan

$$\begin{aligned}
& (\gamma_{12} \cup \gamma_{22})^c(u, v) \\
&= (\gamma_{11} \cup \gamma_{21})(u) \times (\gamma_{11} \cup \gamma_{21})(v) - (\gamma_{12} \cup \gamma_{22})(u, v) \\
&= \gamma_{11}(u) \times \gamma_{21}(v) \\
&\quad \text{karena } \mathbb{Q}_{12}(u, v) = \mathbb{Q}_{22}(u, v) = 0 \\
&= \gamma_{11}^c(u) \times \gamma_{21}^c(v) \quad \text{(Definisi 3.5)} \\
&= (\gamma_{12}^c + \gamma_{22}^c)(u, v) \quad \text{(Definisi 3.6)}
\end{aligned}$$

Dari (a), (b), (c), (d), dan (e) terbukti bahwa

$$\begin{aligned}
& ((\mathbb{Q}_{11} \cup \mathbb{Q}_{21})^c, (\mathbb{Q}_{12} \cup \mathbb{Q}_{22})^c) = (\mathbb{Q}_{11}^c + \mathbb{Q}_{21}^c, \mathbb{Q}_{12}^c + \mathbb{Q}_{22}^c) \\
& ((\gamma_{11} \cup \gamma_{21})^c, (\gamma_{12} \cup \gamma_{22})^c) = (\gamma_{11}^c + \gamma_{21}^c, \gamma_{12}^c + \gamma_{22}^c) \quad \blacksquare
\end{aligned}$$

Teorema 3.11 [2]

Jika G_1 dan G_2 adalah produk graf *fuzzy intuitionistic*, maka perkalian G_1 dan G_2 dinotasikan dengan $G(V_1 \times V_2, E_1 \times E_2)$ adalah produk graf *fuzzy intuitionistic*.

Bukti:

Untuk membuktikan bahwa $G(V_1 \times V_2, E_1 \times E_2)$ adalah produk graf *fuzzy intuitionistic* yaitu dengan menunjukkan bahwa untuk setiap $u_1, v_1 \in V_1$ dan untuk setiap $u_2, v_2 \in V_2$ memenuhi

$$\begin{aligned}
& (\mathbb{Q}_{12} \times \mathbb{Q}_{22})(u_1, u_2, v_1, v_2) \leq (\mathbb{Q}_{11} \times \mathbb{Q}_{21})(u_1, u_2) \times (\mathbb{Q}_{11} \times \mathbb{Q}_{21})(v_1, v_2) \text{ dan} \\
& (\gamma_{12} \times \gamma_{22})(u_1, u_2, v_1, v_2) \leq (\gamma_{11} \times \gamma_{21})(u_1, u_2) \times (\gamma_{11} \times \gamma_{21})(v_1, v_2)
\end{aligned}$$

Untuk setiap $u_1, v_1 \in V_1$ dan untuk setiap $u_2, v_2 \in V_2$, didapatkan

$$\begin{aligned}
& (\mathbb{Q}_{12} \times \mathbb{Q}_{22})(u_1, u_2, v_1, v_2) = \mathbb{Q}_{12}(u_1, v_1) \times \mathbb{Q}_{22}(u_2, v_2) \quad \text{(Definisi 2.4)} \\
&\leq [\mathbb{Q}_{11}(u_1) \times \mathbb{Q}_{11}(v_1)] \times [\mathbb{Q}_{21}(u_2) \times \mathbb{Q}_{21}(v_2)] \quad \text{(Definisi 3.1)} \\
&= [\mathbb{Q}_{11}(u_1) \times \mathbb{Q}_{21}(u_2)] \times [\mathbb{Q}_{21}(u_2) \times \mathbb{Q}_{21}(v_2)] \\
&= (\mathbb{Q}_{11} \times \mathbb{Q}_{21})(u_1, u_2) \times (\mathbb{Q}_{11} \times \mathbb{Q}_{21})(v_1, v_2) \quad \text{(Definisi 2.4)}
\end{aligned}$$

dan

$$\begin{aligned}
& (\gamma_{12} \times \gamma_{22})(u_1, u_2, v_1, v_2) = \gamma_{12}(u_1, v_1) \times \gamma_{22}(u_2, v_2) \quad \text{(Definisi 2.4)} \\
&\leq [\gamma_{11}(u_1) \times \gamma_{11}(v_1)] \times [\gamma_{21}(u_2) \times \gamma_{21}(v_2)] \quad \text{(Definisi 3.1)} \\
&= [\gamma_{11}(u_1) \times \gamma_{21}(u_2)] \times [\gamma_{21}(u_2) \times \gamma_{21}(v_2)] \\
&= (\gamma_{11} \times \gamma_{21})(u_1, u_2) \times (\gamma_{11} \times \gamma_{21})(v_1, v_2) \quad \text{(Definisi 2.4)} \quad \blacksquare
\end{aligned}$$

IV. PENUTUP

Berdasarkan pembahasan yang telah dijelaskan dalam sebelumnya dapat disimpulkan bahwa produk graf *fuzzy intuitionistic* adalah salah satu jenis graf yang merupakan pengembangan dari graf *fuzzy intuitionistic* dimana untuk setiap produk

graf *fuzzy intuitionistic* adalah graf *fuzzy intuitionistic* namun tidak berlaku sebaliknya. Komplemen dari produk graf *fuzzy intuitionistic* adalah produk graf *fuzzy intuitionistic* dan doble komplemen dari produk graf *fuzzy intuitionistic* menghasilkan produk graf *fuzzy intuitionistic* awal. Untuk operasi join dari produk graf *fuzzy intuitionistic* didapatkan bahwa join dari produk graf *fuzzy intuitionistic* adalah produk graf *fuzzy intuitionistic*. Komplemen dari join dua buah produk graf *fuzzy intuitionistic* sama dengan union dari komplemen dua buah produk graf *fuzzy intuitionistic* tersebut dan komplemen dari union dua buah produk graf *fuzzy intuitionistic* sama dengan join dari komplemen produk graf *fuzzy intuitionistic* tersebut. Untuk operasi perkalian dari produk graf *fuzzy intuitionistic* didapatkan bahwa perkalian dari produk graf *fuzzy intuitionistic* adalah produk graf *fuzzy intuitionistic*.

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